Investment, Welfare and the Informativeness of Financial Markets

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Abstract

Asset prices aggregate information about current and future productivity. More information allows firms to make better investment decisions, potentially increasing social welfare. In this paper, I quantify the social value of the information transmitted by the financial system to firms in a real business cycle framework, in an environment with common productivity shocks and temporary, idiosyncratic shocks which inhibit agents' learning. As a benchmark, I first study the value of information in the absence of a financial system. The observation of the permanent component of aggregate productivity has a positive and significant impact on welfare, and an important fraction of the gain comes from the fact that information reduces the long-run risk faced by the household. Next, I include the financial sector as a noisy rational expectations asset market: financial traders observe private signals about the permanent component of productivity, the stock price aggregates the dispersed information of traders, and noise traders prevent full revelation. I find that both reducing the size of the noisy trade or decreasing the dispersion of information among financial traders have negligible effects on welfare, because the ability of firms to learn from the stock price does not change with these parameters. Finally, increasing the precision of the overall information aggregated by the financial system has a positive but modest effect on social welfare.

Keywords: Social value of information, business cycles, learning from prices.

JEL Classification Numbers: E2, E3, D6, D8.

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1 Introduction

Expectations about the future are essential determinants of the current level of consumption and investment. Consumers defer consumption if they expect high returns on savings and firms undertake investment if they expect capital to be significantly more productive in the future. Agents who anticipate changes in productivity are able to achieve improved allocations over time by optimally choosing the intensity and timing of production and investment. The ability to optimally allocate resources over time depends on the quality of information agents have about the future path of productivity. Therefore, more precise information about future productivity is valuable to economic agents. As in Hayek [1945], information contributes to the efficient allocation of investment and potentially improves social welfare.

The financial system plays a key role in providing the real economy with information about future economic conditions. Financial intermediaries acquire and produce information about productivity when evaluating the potential return of financial assets. Financial intermediation is useful because it provides information about investment opportunities, which allows for more efficient allocation of resources (Levine [2005]). In an ideal market, asset prices aggregate information about current and future productivity and convey it to firms and consumers. But, how good is the financial system at transmitting information to the real economy? How valuable is the information about future productivity? How much does the allocation of resources over time improve with this information?

This paper quantifies the social value of information about future aggregate productivity that is collected and transmitted throughout the financial system. News about future aggregate productivity has been shown to be an important source of economic fluctuations (Beaudry and Portier [2006]). News has significant effects on the behavior of investment and output by affecting the timing of investment decisions. Information about future aggregate productivity allows agents to know better the right time to invest, rather than where to invest (in which sector). This information can improve welfare through a better allocation of resources over time. The analysis in this paper allows me to establish the importance of the correct timing of investment and how socially valuable is the information provision function of the financial sector.

To understand how useful the information collected by financial markets is in driving aggregate investment, I build a dynamic framework in which agents can partially learn about the realization of future innovations. I embed a noisy rational expectations asset market in a real business cycle (RBC) model where productivity is the main driver of economic fluctuations. In the model, households consume and save, firms invest in physical capital, and stock prices transmit information about future productivity. The RBC paradigm is well suited for the study of intertemporal choices, and it has been widely used to perform quantitative welfare analysis. Using a standard quarterly parameterization, I evaluate the consequences
on welfare of receiving information about future productivity through stock prices.

First I consider the idealized case in which asset prices transmit information perfectly to the real economy. This is a useful benchmark for studying the value of information in improving the allocation of resources over time. Under a standard parameterization, I find an upper bound to the welfare gains associated with better information about future productivity.

Next, I incorporate a noisy stock market and study the aggregation and transmission of information through the financial system. In the model, informed financial traders observe the level of current aggregate productivity and receive dispersed information about future productivity. When trading, their information is aggregated into the stock price. I study quantitatively the effect that changes in each of the informational features of the stock market have on welfare, and compare them with the benchmark obtained under the ideal case of perfect information transmission.

In the first part of the paper I consider a simple version of the model in which the stock market perfectly aggregates and communicates information to the real economy. Information is exogenous and centralized. In the simple model, the economy admits a representative agent and the allocation of resources and the use of information are efficient. Aggregate productivity is composed of a permanent and a transitory component. Shocks to the permanent component slowly build up and have a strong impact in the future level of productivity, relevant to inform investment decisions. The representative household observes the current realization of productivity, but the transitory shocks prevent the household from recovering the permanent shocks. I compare the welfare attained in an economy in which current productivity is the only information available with the welfare attained in an economy in which the household observes the permanent component as well. Better information about future productivity improves welfare, and under the baseline calibration for CRRA preferences, this better information amounts to a permanent increase of ∼ 0.2% in annual consumption.

This welfare gain is significantly lower than what has been found in the extant literature. For example, Hassan and Mertens [2011] find large gains from the information about future productivity transmitted by the financial system. As many other papers in macroeconomic finance and in the literature of long-run risks, Hassan and Mertens [2011] describe the household preferences with the recursive utility representation of Epstein and Zin [1989]. To shed light on my results, I evaluate the welfare gain attained under Epstein-Zin preferences. In this case, the observation of the permanent component of productivity generates a welfare gain equivalent to a permanent increase of over 2% in annual consumption. This welfare gain is in line with what has been found by the previous literature.

This exercise raises the question of what exactly drives the difference between the CRRA and the Epstein-Zin case. I decompose the welfare gain into two components: (i) the gains due only to the access to better information, fixing allocations, and (ii) the gains due only to the change in the allocation. I find that under Epstein-Zin preferences a negligible fraction of
the welfare gain is explained by a better allocation of resources over time. The observation of
the permanent component of productivity reduces the long-run risk faced by the household,
and this reduction has a very large impact on welfare (an effect associated with the preference
for early resolution of uncertainty, as in Epstein et al. [2014]). Thus, almost all of the welfare
gain is due to the reduction of the anxiety that the household feels about the future, and
an insignificant part is due to actual improvements in the allocation of investment. Since
the purpose of this paper is to assess how useful information is in improving the allocation
of resources over time, I focus on specifications with CRRA preferences to highlight this
mechanism.

In the second part of the paper, I develop a framework in which the financial system aggre-
gates information about future productivity and transmits it to the real sector. Asset prices
are informative but noisy signals. Agents learn from prices, but different shocks confound
the information contained in prices. I embed a simple noisy rational expectations asset market in
the RBC model: financial traders observe private signals about the permanent component of
productivity, the stock price aggregates the dispersed information of traders, and noisy trade
prevents full revelation. Firms extract information from asset prices and use it to produce
and invest. To evaluate the social value of information in this setting, I study how changes
in the informational features of the financial system affect the economy and impact welfare.
To perform the quantitative exercise, I calibrate this model at a quarterly frequency using a
simulated method of moments (SMM) approach.

The financial system is characterized by three information parameters: the degree of
dispersion in the information held by traders, the variance of the noisy trade, and the precision
of the information available to the financial system. The size of the noisy trade and the degree
dispersion of information affect how information is transmitted through the stock price,
but do not affect the overall precision of the information available in the economy. They
have very small effects on macroeconomic aggregates. On the other hand, an increase in the
precision of financial information has more significant effects on macroeconomic moments. In
particular, investment responds more strongly to market signals, increasing the volatility of
its growth rate and the correlation with stock market returns.

This occurs because the stock price conveys information about both current and future
productivity to firms. Consider the case in which the precision of the aggregate financial
information is kept constant. A reduction in the dispersion of information among traders or
a decrease in the size of the noisy trade increases the precision of the information available to
each individual trader. This information includes private signals about future productivity.
In equilibrium, traders respond more strongly to more precise signals. Then, the stock price
depends more on the aggregate signal about the future received by the financial sector and
reflects less information about current productivity. Surprisingly, this does not increase the
ability of firms to learn about the permanent component from the stock price. The reason is
that the level of current productivity is also an informative signal about long-run productivity. The stock price substitutes one source of information for another. The net effect is that the informativeness of the stock price about the permanent component of productivity is approximately constant. Since firms do not learn more about the permanent component from the stock price and the overall precision of the information available to the economy has not changed, there are very small effects on macroeconomic aggregates.

Finally, I study how a change in each information parameter impacts welfare. I find that the dispersion of information and the volatility of noisy trade have a negligible effect on welfare. On the other hand, the overall quality of the information available to the financial system has a positive but modest effect on welfare. It is comparable in size to the welfare gain found in the representative agent case. Moving from the economy under the baseline calibration to an economy in which the aggregate financial signal is completely accurate amounts to a permanent increase of $0.05\%$ in annual consumption.

The contribution of this paper to the literature is fourfold: first, it is the first paper to quantify the macroeconomic effects of the information mediated through the financial system, separating the role of noisy trade, dispersed information and the overall precision of financial information; second, it is the first paper in which the informational content of the stock price is embedded in an RBC framework and depends endogenously on the real economy; third, it decomposes the welfare gain/loss associated with more precise information; finally, this paper allows for a more general information structure in which the permanent component of productivity is never fully learned.

There is a growing effort to study the social value of information in standard macroeconomic frameworks. Angeletos et al. [2011] investigate how information impacts welfare in a static business-cycle framework with monopolistic price-setting. Angeletos and La’O [2011] study optimal monetary policy in a model with physical capital. These papers are concerned with the effect of information on the equilibrium degree of coordination in price setting, but do not include information aggregation and transmission through the financial sector and do not attempt to quantify the effects of welfare.

Hassan and Mertens [2014] embed a canonical noisy rational expectations model within the framework of a business-cycle model. They find small effects of dispersed information on the statistical moments of macroeconomic aggregates, except for the correlation of consumption growth and investment growth, and argue that large welfare gains are possible with Epstein-Zin preferences. In their model, the informativeness of the stock price is detached from the real economy: the signal produced in the stock market is independent from the choices of firms and consumers. Information is dispersed among households, and reducing dispersion amounts to an increase in the precision of the information available to consumers. I find that increasing the overall precision of information has positive effects on welfare as well. In my model, however, the learning of firms from the financial market is not affected by changes in
the dispersion of information or by changes in the size of noisy trade, because adjustments in real decisions feedback to the stock price. The role of this feedback mechanism and the endogenous change in the information content of the stock price has not been studied in a macroeconomic framework before.

Many papers study the aggregation of information through the financial system. Bai et al. [2013] measure the information contained in stock prices and find it has not changed in the past 40 years. They provide microeconomic evidence on the weakness of the information channel. David et al. [2013] study the role of stock prices in allocating resources among different producers. They conclude that firms learn little from financial markets, and turn more to internal sources of information. I contribute to the literature by showing that learning from financial markets has limited effects on an intertemporal, general equilibrium setting. I show that improvements in the ability of the stock price to aggregate information have no effect on macroeconomic aggregates or welfare, embedding the mechanism in a macroeconomic framework.

The paper is organized as follows: Section 2 studies the value of information in a representative-agent economy; Section 3 presents the model with heterogeneous agents and financial trading; Section 4 quantifies the social value of information in the decentralized economy; the last section concludes.

2 The representative-agent economy

In this section I study the social value of information in an economy that admits a representative agent. In particular, information is exogenously given and issues of dispersed information and its aggregation and transmission through prices do not arise. This can be thought as an ideal economy in which asset prices perfectly aggregate and transmit information. Thus, we can focus on the value of information when both the allocation and the use of information are efficient.

2.1 The model

Time is discrete and indexed by \( t \in \{0, 1, 2, \ldots \} \). The economy is inhabited by a representative household endowed with preferences

\[
U = \sum_{t=0}^{\infty} \beta^t \left( \vartheta_t U (C_t - hC_{t-1}) - A_{t+1}^{-\sigma} V (N_t) \right),
\]
The disutility of the labor effort is scaled by the aggregate productivity, $A_t$, to keep the preferences consistent with the existence of a balanced growth path\(^1\). The variable $\vartheta_t$ represents an exogenous demand shock.

The representative agent has access to a production technology given by

$$Y_t = K_{t-1}^\alpha \left( A_t N_t \right)^{1-\alpha},$$

where $Y_t$ is output, $K_{t-1}$ is the stock of installed capital, and $\alpha \in (0, 1)$. The capital stock evolves according to

$$K_t = (1 - \delta) K_{t-1} + \chi_t X_t \left( 1 - \frac{\phi}{2} \left( \frac{X_t}{X_{t-1}} - 1 \right)^2 \frac{X_{t-1}}{X_t} \right),$$

where $X_t$ is gross investment, $\delta \in [0, 1]$ is the depreciation rate, $\phi > 0$ and $\chi_t$ is the capital-embodied technology shock. The output is used only for consumption and investment, and the aggregate resource constraint must hold:

$$Y_t = C_t + X_t.$$

The objective of the household is to maximize expected utility subject to the technological and resource constraints. The level of current productivity, $A_t$, is observed when the representative agent is making consumption and investment decisions. Future productivity is not directly observed, but I will allow the household to obtain partial information about it. The questions of interest are: How valuable is the information about future productivity for the representative agent? How does the value of information depend on preferences?

\subsection*{2.2 A simple example}

In order to illustrate the importance of preferences in determining the usefulness of information, consider the simple example presented in Brock and Mirman [1972]. In particular, assume that utility is logarithmic in consumption ($\sigma = 1$), there is no habit formation ($h = 0$), labor supply is constant, normalized to unity and does not generate disutility ($\psi = 0$), there are no adjustment costs for investment ($\phi = 0$) and capital fully depreciates every period

\footnote{The disutility of labor can be interpreted as the utility cost due to foregone home production. A larger level of aggregate productivity implies that labor becomes more productive when used in home production, and its opportunity cost is larger. Any time spent producing away from home then has a larger utility cost.}
(δ = 1). Furthermore, assume that there are no demand or capital-embodied shocks. Then, the problem of the representative household reduces to

\[
\max_{\{C_t, K_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log (C_t),
\]

subject to

\[
K_{t-1}^\alpha A_t^{1-\alpha} = C_t + K_t.
\]

In this case the problem has a simple analytic solution, and the optimal allocation satisfies

\[
\begin{align*}
C_t &= (1 - \alpha \beta) Y_t, \\
K_t &= \alpha \beta Y_t, \\
Y_t &= K_t^\alpha A_t^{1-\alpha}.
\end{align*}
\]

Consumption and investment are proportional to current output, which only depends on current productivity.

The somewhat surprising fact is that the efficient allocation is completely independent from any information regarding future productivity. The household could not know anything about future productivity, it could perfectly know future productivity, or it could stand anywhere in between, but will always choose the same optimal allocation. The reason is that, in this particular example, the income and substitution effects perfectly offset each other. Suppose that the household expects higher productivity tomorrow. It would like to enjoy more consumption today, due to the larger expected future production. However, the opportunity cost of current consumption increased, because one unit saved today gives a larger return tomorrow. Both effects cancel each other, making the allocation independent from expectations about the future.

It is important to clarify that information might have effects on welfare even if the optimal allocation does not respond to it. To see this, let \( S_t \) denote the information about future productivity available to the household during period \( t \). Consider the value function associated with the household’s problem at period \( t \),

\[
V_t (K_{t-1}, A_t, S_t) = \max_{\{C_{t+j}, K_{t+j}\}_{j=0}^{\infty}} \mathbb{E}_0 \left[ \sum_{j=0}^{\infty} \beta^j \log (C_{t+j}) | K_{t-1}, A_t, A_{t-1}, \ldots, S_t \right].
\]

It can be shown that

\[
V_t (K_{t-1}, A_t, S_t) = \frac{\ln (1 - \alpha \beta) + \frac{\alpha \beta}{1 - \alpha \beta} \ln \alpha \beta}{1 - \beta} + \frac{\alpha}{1 - \alpha \beta} \ln K_{t-1} + \frac{1 - \alpha}{1 - \alpha \beta} \ln A_t + f_S (\ln A_t, S_t),
\]

8
where the function $f_S(\cdot)$ satisfies the recursion

$$f_S(\ln A_t, S_t) = \beta E \left[ f_S(\ln A_{t+1}, S_{t+1}) + \frac{1 - \alpha}{1 - \alpha \beta} \ln A_{t+1} | A_t, A_{t-1}, \ldots, S_t \right].$$

For example, if $A_{t+1}$ is observed in period $t$ and $\ln A_t$ follows a random walk, then $S_t = \ln A_{t+1}$, $E_t[S_{t+1}] = S_t$ and

$$f_S(\ln A_t, S_t) = \beta \frac{1 - \alpha}{(1 - \alpha \beta) (1 - \beta)} S_t.$$

The function $f_S(\cdot)$ is indexed by $S$ because in the general case the functional form depends on the structure of signals $S_t$. Note that in general the time-zero expected value of welfare satisfies

$$E_0 [V_1(\cdot) | A_0, A_{-1}, \ldots, S_0] \neq E_0 [V_1(\cdot) | A_0, A_{-1}, \ldots].$$

Time-zero welfare may differ when the household realizes that the information structure between two otherwise identical economies is different. In particular, the knowledge of the realization of the signal at time zero, $S_0$, is informative and reduces uncertainty about future instantaneous utility streams.

This simple example illustrates two key facts: first, information about future productivity might be useful because it potentially allows for better intertemporal allocations, informing the choice between consumption and investment. Therefore, how useful this information actually is depends strongly on income and substitution effects. Second, even if the optimal allocation does not change with the precision of the information about future productivity, the present value of the utility perceived by the representative household at time zero can change. The reason is that the household evaluates future streams of utility conditioning on currently available information. If the household is sufficiently risk-averse, information about the future reduces the perceived risk of future utility and increases current welfare, even if the allocation remains unchanged.

### 2.3 Productivity and information

In order to study the effect of information about future productivity on welfare and to highlight the main mechanisms at play, we need to specify a simple, parametric process for productivity. Following Blanchard et al. [2013], I assume that the aggregate productivity, $a_t \equiv \ln (A_t)$, is given by

$$a_t = z_t + v_t,$$

where the permanent component $z_t$ follows the unit root process

$$\Delta z_t = \rho \Delta z_{t-1} + \varepsilon_t,$$
and the transitory component follows the stationary process

\[ v_t = \rho v_{t-1} + \mu_t, \]

for \( \rho \in [0, 1] \). The innovations to the components of productivity follow Gaussian distributions, \( \varepsilon_t \sim N\left(0, \sigma^2_{\Delta a} (1 - \rho)^2\right) \) and \( \mu_t \sim N\left(0, \sigma^2_{\Delta a} \rho\right) \), where \( \sigma^2_{\Delta a} \) is the variance of \( \Delta a_t = a_t - a_{t-1} \). The constraints imposed on the variances of the innovations \( \varepsilon_t \) and \( \mu_t \) imply that productivity follows a random walk when considered as a univariate time series,

\[ E[a_t|a_{t-1}, a_{t-2}, \ldots] = a_{t-1}. \]

This parameterization of the productivity process allows for a simple characterization of the partial information case, in which the only information available to the household is the sequence of current and past realizations of \( a_t \). That is, the information set under partial information is defined recursively as

\[ \mathcal{I}_t = \mathcal{I}_{t-1} \cup \{a_t\}, \]

and the best forecast for future productivity, at every horizon, is the current realization of productivity.

In this setting, full information corresponds to the case in which the current and past realizations of the permanent component \( z_t \) are also observed. The full information set is

\[ \mathcal{I}^{full}_t = \mathcal{I}^{full}_{t-1} \cup \{a_t, z_t\}. \]

The realization of the permanent component \( z_t \) is informative about future productivity, because it allows to separate transitory from permanent shocks. Consider the long-run expectation about productivity under partial and full information:

\[ \lim_{j \to \infty} E[a_{t+j} | \mathcal{I}_t] = a_t \quad \text{and} \quad \lim_{j \to \infty} E_t \left[a_{t+j} | \mathcal{I}^{full}_t\right] = \frac{z_t - \rho z_{t-1}}{1 - \rho}. \]

An innovation in the transitory component, \( \mu_t \), is reflected in the level of current productivity \( a_t \), but its effect disappears in the long run. Under partial information, the household attributes all its effect to the permanent component and the long-run expectation moves one-to-one with \( \mu_t \). However, when the household also observes the permanent component, it correctly keeps the long-run expectation unchanged. Finally, note that the observation of \( z_t \) is more useful when the permanent component is more persistent: the weight given to \( z_t \) in the expectations about future productivity increases as \( \rho \) moves towards one.
2.4 The CRRA case

The problem faced by the representative household admits an explicit, analytic solution only in few particular cases. Thus, a numerical solution is required to analyze the welfare properties of the economy. I solve the model with a perturbation method, using a third-order approximation around the deterministic steady state. The parameters of the representative-agent economy are conventional and are taken from the relevant literature. A period in the model corresponds to a quarter. The discount factor is set as $\beta = 0.99$, the share of capital in national income is set as $\alpha = 0.33$, and the depreciation rate is $\delta = 0.025$. In the baseline calibration, I set $\sigma = 2$, $\eta = 1$, $h = 0.3$ and $\phi = 0.6$. Finally, the parameter $\psi$ is chosen to obtain $N = \frac{1}{3}$ in steady state. The baseline calibration is summarized in Table 1. The parameters for habit formation and adjustment cost are obtained from the estimation of the full model that will be introduced in the next section. The standard deviation of the growth rate of productivity, $\sigma_{\Delta a}$, corresponds to the quarterly volatility of the utilization-adjusted TFP time-series constructed by Fernald [2012].

Table 1: Calibration: representative agent, CRRA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ Production function</td>
<td>0.33</td>
</tr>
<tr>
<td>$\beta$ Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\delta$ Depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\sigma$ Risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>$\eta$ Inv. Frisch elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$h$ Habit</td>
<td>0.3</td>
</tr>
<tr>
<td>$\phi$ Adjustment cost</td>
<td>0.6</td>
</tr>
<tr>
<td>$\sigma_{\Delta a}$ St. dev. productivity growth</td>
<td>0.85%</td>
</tr>
</tbody>
</table>

What is the social value of having more precise information about the permanent component of productivity? To answer this question, we need first to define a welfare criterion. I take the expected value, at time zero, of the discounted sum of utility, under the assumption that the economy starts at the deterministic steady state. To determine the social value of information, I compare the value of this welfare criterion in the economy under full information with the value in the economy under partial information.

To express the changes in welfare in meaningful units, I consider the permanent stationary consumption subsidy/tax required to keep the household indifferent between facing the economy with full information or living in the economy with partial information. That is, by which percentage should yearly consumption permanently increase/decrease in the economy under partial information to achieve the same welfare level as in the economy under full information? Or equivalently, what is the minimum permanent compensation in consumption
required for the household to be willing to give up full information?

I find that the observation of the permanent component of productivity is welfare improving. Under the baseline calibration, for a reasonable value of the persistence parameter, $\rho = 0.9$ (as in Blanchard et al. [2013]), the availability of the additional information is equivalent to receiving a permanent increase in annual consumption of 0.21%.

It is worth pointing out that the annual consumption equivalent gain of $\sim 0.2\%$ is economically significant. A household consuming on average $50,000 a year must receive a permanent increase of $100 in its consumption to be willing to give up its information about future productivity. In contrast, according to Lucas [1987], in a framework similar to ours, the household would be willing to pay only $10 to eliminate all fluctuations associated with the business cycle.²

The first panel on Figure 1 presents the consumption equivalent compensation as a function of the persistence parameter of productivity, $\rho$, for different degrees of risk-aversion. In particular, for any $\sigma > 1$, the representative household is better off in the economy with full information: the subsidy required to keep her indifferent is positive. When $\sigma = 1$ the effect of information on welfare is very small for any $\rho$, just as in the example by Brock and Mirman [1972]. Importantly, the consumption equivalent compensation is increasing in the degree of risk aversion. For a plausible degree of persistence of productivity, $\rho = 0.9$, the compensation goes from almost zero percent under log utility, to 0.58% of annualized consumption when $\sigma = 3$. The positive relation between risk aversion and welfare gain arises because better information about future productivity reduces the risk associated with investment. More risk averse households value more the reduction in risk.

The compensation is increasing in the degree of persistence. This is intuitive: a larger $\rho$ implies that the permanent component is more important in determining future values of productivity. The household cares about its future wealth, which is determined by future productivity. Then any information regarding the permanent component is more relevant and useful when $\rho$ is large. For example, under the baseline calibration, the consumption compensation ranges from 0.08% for a moderate degree of persistence of $\rho = 0.75$, to 0.21% when $\rho = 0.9$.

Note that the beneficial effects of more precise information are tightly linked to risk aversion, and depend less on considerations of intertemporal substitution. One way to see this fact is to study how the compensation changes with the investment adjustment cost, parameterized by $\phi$. In the model, the only variable available to smooth out consumption across time is investment. A large adjustment cost, however, limits the ability of the household to freely adjust investment in response to temporary shocks. How important is information on

²The welfare cost of the business cycle is approximately $\frac{1}{2}\sigma Var[c_t]$, and in annualized terms, using a quarterly variance of $Var[c_t] = 0.007^2$, it amounts to 0.02%. This is one order of magnitude less than the welfare cost of not observing the permanent component of productivity.
determining the timing of consumption and investment is thus parameterized by $\phi$, the scale parameter of the investment adjustment cost. The lower panel on Figure 1 shows the consumption equivalent compensation as a function of the persistence parameter of productivity, $\rho$, for different degrees of $\phi$. The compensation is decreasing with $\phi$. The reason is that it is easier for the household to take advantage of more precise information when the adjustment cost is low. However, the change in the equivalent compensation is relatively small. For $\rho = 0.9$, the annual permanent compensation moves from 0.22% when $\phi = 0.25$ to 0.17% when $\phi = 5$. Thus, multiplying $\phi$ by twenty reduces the permanent compensation only by a few basis points. This is in sharp contrast with the large effects observed when changing risk-aversion.
A decomposition of the welfare gains

The observation of the permanent component of productivity is useful, and it amounts to a permanent increase of \( \sim 0.2\% \) of annual consumption equivalent in the representative-agent economy under CRRA preferences. But, what explains the welfare gains?

Theoretically, there are two forces driving the change in welfare due to changes in information: first, the optimal allocation chosen by the household is different under different information sets; second, the expectation operator used to evaluate future streams of utility is different. The purpose of this section is to decompose the welfare gains into a component due only to the change in the expectation operator, and another component related to the change in the allocation.

Let us consider what is the welfare gain/loss due only to the availability of additional information. To do so, it is necessary to control for the effect of the change in the optimal allocation. In particular, I compare the welfare level attained under partial information to the welfare level attained when the household has full information but follows the decision rules that are optimal under partial information. That is, I keep the decision rules found under partial information but evaluate the welfare function under full information.

Let \( C(K_{t-1}, X_{t-1}, A_t, A_{t-1}) \) and \( N(K_{t-1}, X_{t-1}, A_t, A_{t-1}) \) denote the optimal decision rules for normalized consumption and labor followed under partial information. Consumption and labor depend on observable state variables: the stock of installed capital, the level of past
investment and current and past aggregate productivity. Let

\[ \hat{V}_t(K_{t-1}, X_{t-1}, A_t, A_{t-1}) \]

\[ = U(C(K_{t-1}, X_{t-1}, A_t, A_{t-1})) - V(N(K_{t-1}, X_{t-1}, A_t, A_{t-1})) \]

\[ + \beta \left( \frac{A_t}{A_{t-1}} \right)^{1-\sigma} E\left[ \hat{V}_{t+1}(K_t, X_t, A_{t+1}, A_t) \mid K_{t-1}, X_{t-1}, A_t, A_{t-1} \right]. \]

denote the normalized value function (i.e., the discounted sum of expected utility) at time \( t \). The expected value of welfare under imperfect information at time zero is given by

\[ W_0 = E\left[ \hat{V}_1(K_0, X_0, A_1, A_0) \mid K_0 = K, X_0 = X, A_0 = 1, A_{-1} = 1 \right], \]

where it is assumed that the initial conditions correspond to the deterministic steady state of the economy.

Let \( \hat{V}_{t}^{\text{dec}} \) denote the normalized sum of the discounted expected utility, evaluated under full information, when the allocation is fixed and follows the decision rules that were optimal under partial information. It satisfies the recursion

\[ \hat{V}_{t}^{\text{dec}}(K_{t-1}, X_{t-1}, A_t, A_{t-1}, Z_t, Z_{t-1}) \]

\[ = U(C(K_{t-1}, X_{t-1}, A_t, A_{t-1})) - V(N(K_{t-1}, X_{t-1}, A_t, A_{t-1})) \]

\[ + \beta \left( \frac{A_t}{A_{t-1}} \right)^{1-\sigma} E\left[ \hat{V}_{t+1}^{\text{dec}}(K_t, X_t, A_{t+1}, A_t, Z_{t+1}, Z_t) \mid K_{t-1}, X_{t-1}, A_t, A_{t-1}, Z_t, Z_{t-1} \right]. \]

The expected value of welfare attained when the household has full information but follows the decision rules that are optimal under partial information is given by

\[ W_0^{\text{dec}} = E\left[ \hat{V}_1^{\text{dec}}(K_0, X_0, A_1, A_0, Z_1, Z_0) \mid K_0 = K, X_0 = X, A_0 = 1, A_{-1} = 1, Z_0 = 1, Z_{-1} = 1 \right]. \]

The welfare decomposition presented in this Section compares the values attained by \( W_0 \) and \( W_0^{\text{dec}} \).

Figure 2 illustrates the decomposition of the welfare gain for the baseline calibration. A significant fraction of the welfare gain is explained by the change in the expectation operator (keeping the decision rules fixed). Under the CRRA preferences, approximately 70% of the welfare gains are explained purely by a more efficient allocation of resources.

In this exercise, we are comparing two different economies starting from the deterministic steady state. The agents recognize that the information structure is different between the economies, and they expect it to be different always. The signal structure impacts the value function because the representative household internalizes the fact that the stream of future utility will be evaluated using different information sets. In some sense, at the beginning of period zero, the household uses a different prior to evaluate the future, and this makes the
value function dependent on the information structure considered.

2.6 The Epstein-Zin case

Recursive utility preferences are commonly used in asset-pricing and macro-finance models. In particular, the specification proposed by Epstein and Zin [1989] has proven fruitful in both theoretical and applied work. The main appeal of Epstein-Zin preferences is that they allow to disentangle the degree of intertemporal substitutability from the attitude towards risk aversion. In particular, the degree of risk aversion can be considerably increased to obtain significant risk premia while keeping a high intertemporal substitution.

We learned from the CRRA preferences that the degree of risk aversion is positively associated with the welfare gains attained from observing the permanent component of productivity. Also, we concluded that intertemporal substitution plays a minor role in the CRRA case. It seems natural then to extend the analysis to the Epstein-Zin specification, where both effects can be cleanly separated and analyzed.

The economic environment is the same as the one presented in Subsection 2.1. The only difference is that the household is endowed with Epstein-Zin preferences. In particular, the representative household solves the problem

\[ V_t = \max_{C_t,N_t,X_t,K_t} \left( (1 - \beta) \tilde{C}_t^{1-\frac{1}{\Psi}} + \beta E_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\Psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\Psi}}}, \]

subject to

\[ \tilde{C}_t = C_t^\alpha (A_{t-1} (1 - N_t))^{1-\alpha}, \]
\[ Y_t = K_t^\alpha (A_t N_t)^{1-\alpha}, \]
\[ K_t = (1 - \delta) K_{t-1} + X_t \left( 1 - \frac{\phi}{2} \left( \frac{X_t}{X_{t-1}} - 1 \right)^2 \frac{X_{t-1}}{X_t} \right), \]
\[ Y_t = C_t + X_t. \]

In this specification, the elasticity of intertemporal substitution is given by \( \frac{1}{\Psi} > 0 \), and the degree of risk aversion is measured by \( \gamma > 0 \).

The technological parameters are the same as in the previous section. Following Hassan and Mertens [2014], I set \( \Psi = 2 \). In the baseline calibration, I choose a moderate level of risk aversion, \( \gamma = 4 \). Finally, the elasticity parameter \( \phi \) is chosen to obtain \( N = \frac{1}{5} \) in steady state.

The top panel in Figure 3 presents the consumption equivalent compensation as a function of the persistence parameter of productivity, \( \rho \), for different degrees of risk-aversion. As in the CRRA case, the household is better off when it observes the permanent component of productivity, and the welfare gains are increasing in the degree of risk aversion. However, the
welfare gains are almost one order of magnitude larger under Epstein-Zin preferences than in the CRRA case: for $\rho = 0.9$, the consumption equivalent compensation ranges from 0.38% when $\gamma = 1.5$, 1.74% when $\gamma = 4$, to 3.94% when $\gamma = 8$.

Figure 3: Epstein-Zin - Permanent consumption compensation

The lower panel in Figure 3 shows the consumption equivalent compensation as a function of the persistence parameter of productivity, $\rho$, for different degrees of intertemporal substitution. The consumption compensation is decreasing in the elasticity of intertemporal substitution. The reason is that it is less painful for the household to take advantage of more precise information when the utility cost of substituting consumption intertemporally is low. However, the response of the compensation to changes in $\Psi$ is relatively small. For example, the consumption compensation ranges from 1.74% for $\Psi = 2$ to 1.13% for $\Psi = \frac{1}{4}$ when $\rho = 0.9$. 

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The effect of intertemporal substitution is significantly weaker than the effect of risk-aversion, just as in the CRRA case.

Figure 4 illustrates the decomposition of the welfare gain for the baseline calibration. In sharp contrast to the CRRA case, a negligible fraction of the welfare gain is explained by a more efficient allocation under Epstein-Zin preferences. This result is related to the preference for early resolution of uncertainty that is characteristic of the recursive utility representation when $\gamma > \frac{1}{\Psi}$. The representative agent values any information about the future, and information about long-run risk is very valuable in the Epstein-Zin framework. The observation of the permanent component of productivity partially resolves the uncertainty associated with the long-run value of productivity. As illustrated by Epstein et al. [2014], under commonly used calibrations, the household would be willing to pay very large amounts to resolve long-run risk. In our context, this implies that the information about the long-run component of productivity is very valuable, just because it resolves long-run uncertainty early. Therefore, almost all the gain in welfare is due to the reduction in risk, and not to the improvement due to the choice of a different allocation.

Finally, I compare the level of the consumption compensation that is due only to the change in the optimal allocation, between the CRRA economy and the Epstein-Zin economy. Figure 5 presents the result. The consumption compensation is almost one order of magnitude smaller in the CRRA economy, but close to 30% of the welfare gain is due to the achievement of a better allocation. For $\rho = 0.9$, a better allocation generates 0.15% of annualized consumption gain (out of a total of 0.21%). In contrast, the large welfare gain observed in the Epstein-Zin case is almost completely due to the reduction in long-run risk, and the effect of changes in the optimal allocation in the consumption compensation is extremely small. For $\rho = 0.9$, the
change in the allocation generates only 0.005% of annualized consumption gain (out of a total of 1.74%) in the Epstein-Zin case.

Figure 5: Welfare gain explained by change in allocation - Baseline calibration

![Graph showing consumption compensation due to change in the allocation - annualized percent.]

We need to be careful when interpreting the social value of the information provided by the financial sector. The standard narrative dictates that information is useful because it improves the allocation of resources over time (Hayek [1945]). However, under recursive utility preferences, that is not the case: information has value because it reduces the anxiety the consumer feels about the future. It is not enough to report that there is a social welfare gain due to more precise information. It is necessary to understand what causes the gain. Since the purpose of this paper is to assess how useful information is in improving the allocation of resources over time, I keep working with CRRA preferences from now on.

3 The decentralized economy

In this section I present a decentralized, competitive economy that incorporates a simple financial sector.

3.1 The environment

The economy is inhabited by a representative household, a (unit-measure) continuum of firms, a continuum of informed financial traders for each firm, and noise traders. The household is composed of a consumer and a unit-measure continuum of workers, one worker for each firm. The firms hire labor from the household and combine it with their own physical capital to produce the final good. Firms face heterogeneous productivity shocks and take decisions
on production and investment under imperfect information. The financial sector trades on outstanding shares of each firm, based on private information regarding current and future aggregate productivity. The only role of the financial traders is to aggregate information about the current and future state of the economy and to transmit it to firms through the share/stock price. This simplification will allow us to focus on the effects of the information channel, abstracting from other important functions that the financial system might play.

Aggregate productivity is driven by a permanent shock and a transitory shock. The components are not observed separately. The permanent component determines the long-run behavior of aggregate productivity, which is relevant for investment and saving decisions. Agents observe signals about the permanent component, but they never fully learn its value. Eventually, market activity reveals aggregate productivity, which is learned by all agents. However, the temporary component prevents learning the permanent shocks from the observation of productivity, creating a signal extraction problem.

Time is discrete and indexed by $t \in \{0, 1, 2, \ldots \}$. Each period has two stages. All agents enter the first stage holding common beliefs about the permanent component of productivity. During the first stage, workers and firms meet and production and investment occur in local, firm-specific markets. Each firm, indexed by $i \in [0, 1]$, and its corresponding worker and financial traders, learn the local level of productivity, observe the stock price of the firm and update their beliefs. Furthermore, each informed trader observes the level of current aggregate productivity and receives information about future productivity. When trading, the private information of traders is aggregated into the stock price, which becomes an informative signal useful to guide production and investment decisions. After production, investment and financial trading are completed, workers return home and the economy enters the second stage. The consumer learns all the information that was previously dispersed, updates her beliefs, goes shopping and decides on consumption and saving. At the end of the period, all agents observe aggregate productivity and learn the updated belief of the consumer, which becomes their common belief at the start of the next period.

### 3.2 The firms

There is a continuum of firms distributed uniformly on the unit interval. The output of firm $i$ during period $t$ is given by

$$Y_{i,t} = (U_{i,t}K_{i,t-1})^\alpha (N_{i,t}A_{i,t})^{1-\alpha},$$

where $A_{i,t}$ is the productivity level of firm $i$, $K_{i,t-1}$ is the stock of capital owned by firm $i$, $U_{i,t}$ is the degree of utilization of the capital stock and $N_{i,t}$ are the hours of labor hired. The
capital stock of the firm evolves according to

\[ K_{i,t} = (1 - \delta) K_{i,t-1} + \chi_t X_{i,t} \left( 1 - \frac{\phi}{2} \left( \frac{X_{i,t}}{X_{i,t-1}} - 1 \right)^2 \frac{X_{i,t-1}}{X_{i,t}} \right), \]

where \( \chi_t \) is the capital embodied shock, \( X_{i,t} \) is the amount of final good purchased for gross investment, \( \delta \in [0, 1] \) is the depreciation rate and \( \phi > 0 \). The shock \( \chi_t \) follows an AR(1) process, is common across firms and its realization is known by everyone at the beginning of the period. Its purpose is to generate additional volatility in the growth rate of investment.

The current profit of the firm is given by

\[ D_{i,t} = Y_{i,t} - W_{i,t} N_{i,t} - X_{i,t} - c \frac{U_{i,t}^{1+\varsigma}}{1+\varsigma} K_{i,t-1}, \]

where \( W_{i,t} \) is the wage rate, \( c \geq 0 \) and \( \varsigma > 0 \). Current profit is the revenue net of the wage bill, gross investment and the utilization cost of capital.

The ownership of the firm is divided in a continuum of outstanding shares of measure one. Financial traders on firm \( i \) trade and own the shares. However, the financial traders act on behalf of the household. As such, the ultimate claimant for all profits or losses made by the firm is the household. Therefore, the objective of the firm is to maximize the expected value of the discounted sum of profits, where the discount factor is given by the marginal utility of consumption, \( \Lambda_t \). The problem faced by firm \( i \) during period \( t \) is

\[ \max \left\{ \{N_{i,t+1}, U_{i,t+1}, K_{i,t+1}, X_{i,t+1}\}_{l=0}^{\infty} \right\} \ E_{i,t} \left[ \sum_{l=0}^{\infty} \beta^l \Lambda_{t+l} D_{i,t+l} \right], \]

subject to the production function and the capital accumulation equation. Here, \( E_{i,t} \) denotes the expectation operator conditional on the information available to firm \( i \) at the beginning of period \( t \), \( E_{i,t} \) denotes the expectation operator conditional on the information available to firm \( i \) at the beginning of period \( t \), \( E_{i,t} \) denotes the expectation operator conditional on the information available to firm \( i \) at the beginning of period \( t \), \( E_{i,t} \) denotes the expectation operator conditional on the information available to firm \( i \) at the beginning of period \( t \).

Let \( S_t \) denote the information publicly available at the beginning of period \( t \). Firm \( i \) observes \( S_t \), which includes the past realizations of aggregate consumption, aggregate investment and aggregate productivity, and the common belief about the level of the permanent component of productivity, formed with the information available at \( t - 1 \). The firm also observes its own productivity level, \( A_{i,t} \), and its own stock price, \( Q_{i,t} \). Then, the information set of firm \( i \) is given by

\[ \mathcal{I}_{i,t} = \mathcal{I}_{i,t} \cup \{A_{i,t}, Q_{i,t}, S_t\}. \]

I assume that firm-specific productivity, \( a_{i,t} \equiv \ln(A_{i,t}) \), follows the process

\[ a_{i,t} = a_t + \xi_{i,t}, \]
where $a_t$ is the aggregate component of productivity and $\xi_{i,t}$ is the firm-specific component of productivity. The specific productivity shock, $\xi_{i,t}$, follows the AR(1) process

$$
\xi_{i,t} = \rho \xi_{i,t-1} + \omega_{i,t},
$$

where $\omega_{i,t} \sim N(0, \sigma^2_{\omega})$ is independent of any other shock, i.i.d. over time and satisfies $\int_0^1 \omega_{i,t} di = 0$.

The productivity level of firm $i$ is a noisy signal about the current level of aggregate productivity. Also, since $a_{t-1}$ is observed, $\xi_{i,t-1}$ is known by the firm.

### 3.3 The household

There is a representative household composed of a consumer and a continuum of workers of measure one. At the beginning of each period $t$, the household sends worker $i$ to supply labor for firm $i$ before any market activity has taken place. The workers decide on their labor effort before observing the realization of any aggregate shock, and their labor supply does not reveal any new information to the firm. On the other hand, the wage rate $W_{i,t}$ and the stock price $Q_{i,t}$ are observed by the worker, who learns the productivity level of the firm. Therefore, worker $i$ and firm $i$ share the same information set.

At the end of period $t$, the workers return home and the consumer learns all the information that was dispersed. Then she decides on consumption and saving.

The representative household has preferences

$$
U_t = E_t \left[ \sum_{l=0}^{\infty} \beta^l \left\{ \frac{\vartheta_{t+l} (C_{t+l} - hC_{t+l-1})^{1-\sigma}}{1 - \sigma} - A_t^{1-\sigma} \psi \int_0^1 N_{i,t+l}^{1+\eta} di \right\} \right],
$$

where $C_t$ denotes consumption, $\sigma, \psi, \eta, h \geq 0$ and $\vartheta_t$ is a shock to the marginal utility of consumption. The shock $\vartheta_t$ follows an AR(1) process and its realization is known by everyone at the beginning of the period. Its purpose is to generate additional volatility in the stochastic discount factor.

The household controls the activities of the financial traders and receives their aggregate profit as a lump sum transfer. Finally, it has access to a bond that pays the risk-free gross interest rate $R_t$. Therefore, the budget constraint is

$$
C_t + B_t = R_t B_{t-1} + \int_0^1 W_{i,t} N_{i,t} di + \Pi_t,
$$

where $B_t$ is the holding of bonds and $\Pi_t$ is the transfer received from the financial traders.
3.4 The stock market

The structure of the stock market follows the noisy rational expectations paradigm in the spirit of Grossman and Stiglitz [1980]. For each firm \( i \) there is unit-measure continuum of informed traders, indexed by \( j \in [0, 1] \). Financial traders exchange the unit-measure of outstanding stock on the firm at the price \( Q_{i,t} \). The asset is a claim on current and future dividends (firm’s profits), \( D_{i,t} \). The current profit of trader \( j \) on firm \( i \) during period \( t \) is

\[
\Pi_{j,i,t} = B_{j,i,t-1} Q_{i,t} + B_{j,i,t} (D_{i,t} - Q_{i,t}) - \frac{\tau}{2} (B_{j,i,t} - 1)^2 K_{i,t-1},
\]

where \( B_{j,i,t} \) is the holding of shares by trader \( j \). Traders are subject to a portfolio adjustment cost, which makes profits concave in \( B_{j,i,t} \) and helps to generate a demand for shares that responds to the asset price.

Financial traders are controlled by the household and transfer their current profit to the consumer. The objective of trader \( j \) is to maximize the expected value of the discounted sum of profits, where the discount factor is given by the marginal utility of consumption, \( \Lambda_t \). The problem faced by trader \( j \) during period \( t \) is

\[
\max \left\{ B_{j,i,t+1} \right\}_{l=0}^{\infty} E_{j,i,t} \left[ \sum_{l=0}^{\infty} \beta^l \Lambda_{t+l} \Pi_{j,i,t+l} \right],
\]

where \( E_{j,i,t} [\cdot] \) denotes the expectation operator conditional on the information available to trader \( j \).

Trader \( j \) on firm \( i \) observes the information publicly available at the beginning of period \( t \), \( S_t \), the stock price of the firm, \( Q_{i,t} \), a private signal about the permanent component of aggregate productivity, \( s_{j,i,t} \), the current level of firm-specific productivity \( A_{i,t} \), and the aggregate productivity, \( A_t \).

The stock market is also inhabited by uninformed, noise traders. They buy and sell the share without any knowledge about economic conditions. In particular, the demand of the noise traders is exogenously given by

\[
B_{t,t}^{\text{noise}} = \exp(e_{t,t}^{\text{noise}}) - 1,
\]

where \( e_{t,t}^{\text{noise}} \sim N(0, \sigma_{\text{noise}}^2) \), and \( \int_{0}^{1} e_{t,t}^{\text{noise}} \, di = 0 \). The only purpose of allowing for uninformed traders is to introduce uncorrelated noise in the stock market, thus preventing the price from becoming perfectly revealing.
3.5 Productivity and information

As in Subsection 2.3, I assume that the aggregate productivity, \( a_t \equiv \ln (A_t) \), consists of a permanent component, \( z_t \), and a transitory component, \( v_t \). Specifically,

\[
a_t = z_t + v_t,
\]

where the permanent component \( z_t \) follows the unit root process

\[
\Delta z_t = \rho_z \Delta z_{t-1} + \varepsilon_t,
\]

and the transitory component follows the stationary process

\[
v_t = \rho_v v_{t-1} + \mu_t,
\]

with \( \rho_z, \rho_v \in [0, 1] \). The innovations to the components of productivity follow Gaussian distributions, \( \varepsilon_t \sim N \left( 0, \sigma_\varepsilon^2 \right) \), \( \mu_t \sim N \left( 0, \sigma_\mu^2 \right) \), are independent of any other shock, and i.i.d. over time.

The main source of uncertainty in the model is the permanent component of aggregate productivity. No agent directly observes \( z_t \). However, the informed financial traders receive noisy signals about it:

\[
s_{j,i,t} = z_t + \varepsilon_{j,i,t} + e_{t}^f,
\]

where \( e_{t}^f \sim N \left( 0, \sigma_f^2 \right) \) is correlated noise in the private signals, and \( \varepsilon_{j,i,t} \sim N \left( 0, \sigma_{disp}^2 \right) \) is idiosyncratic noise such that \( \int_0^1 \varepsilon_{j,i,t}d_j = 0 \) for all \( i \in [0, 1] \).

Let

\[
s^f_t = z_t + e^f_t
\]

denote the aggregate signal received by the financial sector. That is, \( s^f_t = \int_0^1 s_{j,i,t}d_j \) is the cross-sectional average of the dispersed information received by the financial traders. In equilibrium, this aggregate signal is partially transmitted through stock prices, and it summarizes the knowledge of the financial sector about \( z_t \).

At the end of each period, the consumer learns all the information that was previously dispersed. Therefore, she observes the aggregate productivity, \( a_t \), and the aggregate signal received by the financial sector, \( s^f_t \). Due to the transitory shock, the consumer cannot back out the permanent shock from the observed aggregate productivity. Thus, she has to solve a signal extraction problem.

Let

\[
z_{t|t} = E \left[ z_t | a_t, s^f_t, a_{t-1}, s^f_{t-1}, \ldots \right]
\]

be the belief about the current level of \( z_t \), given the current and past realization of the
aggregate signals. The consumer decides on consumption and saving using her updated beliefs. All the agents in the economy observe the realization of consumption at the end of the period, and they learn the belief \( z_{t|t} \). Since the information of the consumer is superior to the dispersed and incomplete information of firms and traders, every agent in the economy begins the next period holding the same belief \( z_{t|t} \).

The fact that all agents in the economy enter each period with common beliefs imply that we only need to keep track of \( z_{t|t} \) to describe the intertemporal evolution of the beliefs held by all the agents in the economy. This greatly simplifies the solution to the model. Finally, the Gaussian structure of the productivity process and the signals allow the household to update its own beliefs through standard Kalman filtering.

### 3.6 Equilibrium and solution

Given a time path of the exogenous innovations, a stationary equilibrium in the decentralized economy is a sequence of quantities, prices and signals, such that:

1. the supply of labor of worker \( i \) maximizes the expected value of utility, given the wage rate \( W_{i,t} \) and the observation of the firm-specific productivity level and the stock price,

2. the demand for labor and the choice of utilization rate and capital investment by firm \( i \) maximize the expected value of profits, given the observation of the firm-specific productivity level and the stock price,

3. the demand of trader \( j \) for the stock of firm \( i \) maximizes her expected profit, given the observation of aggregate productivity, the private signal about the permanent component of productivity and the stock price,

4. consumption and bond holdings maximize the expected utility of the household, given the observation of all the aggregate signals,

5. all technological constraints hold and markets clear.

I compute the first-order conditions that characterize the equilibrium in Appendix B.1. It is worth noticing that the cross-sectional distribution of capital and investment across firms constitute a state variable in the model. The reason is that current investment and production depend on past investment and capital. To forecast future income, the agents in the economy must understand how the cross-sectional distribution evolves over time. This complicates the solution of the exact model and precludes the use of non-linear perturbation methods.

To concentrate in the effect of information on welfare, while keeping firm heterogeneity, I adopt the usual practice in the RBC-DSGE literature, and log-linearize the equations characterizing the behavior of the stationary version of the model about the deterministic steady-state. Then I find the symmetric linear rational expectations equilibrium, taking the
precision of the signals as given, for the log-linearized version of the model. This approach has three obvious advantages: first, in the linear-Gaussian equilibrium, the cross-sectional distribution of firm-specific characteristics is normal, with a time invariant variance. Thus, it is only necessary to keep track of the cross-sectional mean of capital and investment, which coincide with the corresponding macroeconomic aggregates. Second, all firms and traders follow symmetric decision rules, and thus each class of agent can be treated as a “representative” of its kind. All heterogeneity is relegated to value taken by their respective state vector. Third, the equilibrium stock price is a linear function of state variables and shocks, and as such it becomes a linear-Gaussian signal. The signal extraction problem of firms and traders then has a straightforward solution.

In practical terms, I follow a “guess and verify” approach. First, I define the state variables, given by

\[
\begin{align*}
\theta_t &= \begin{bmatrix} a_{t-1} & a_{t-2} & c_{t-1} & k_{t-1} & x_{t-1} & \chi_t & \vartheta_t \end{bmatrix}', \\
\theta_{i,t} &= \begin{bmatrix} k_{i,t-1} & x_{i,t-1} & \xi_{i,t-1} & a_{i,t} \end{bmatrix}', \\
s_t &= \begin{bmatrix} a_t & s_f^t \end{bmatrix}', \\
\theta_b^b &= \begin{bmatrix} z_{t-1|t-1} & v_{t-1|t-1} & z_{t-2|t-1} \end{bmatrix}'.
\end{align*}
\]

The vector \(S_t = \begin{bmatrix} \theta_t & \theta_b^b \end{bmatrix}\) is observed by all agents in the economy and constitutes the aggregate public information available at the beginning of each period. It includes the past realization of aggregate variables, current aggregate shocks not related to productivity, and common beliefs. The vector \(\theta_{i,t}\) denotes firm-specific information, and it is observed by workers and traders in sector \(i\). Finally, \(s_t\) denotes the vector of aggregate signals, observed by the consumer.

Given the state variables, I guess a linear solution for a minimal set of forward looking variables:

\[
\begin{bmatrix} c_t \\ q_{i,t} \\ \varphi_{i,t} \\ x_{i,t} \end{bmatrix} = \begin{bmatrix} A & B & C & D \\ & & & & D \\ & & & & D \\ & & & & & G \\ & & & & & & & & & & G \end{bmatrix} \begin{bmatrix} \theta_t \\ \theta_{i,t} \\ s_t \\ \theta_b^b \end{bmatrix} + \begin{bmatrix} C s_t & + & D e_{i,t}^{\text{noise}} & + & G \theta_b^b \end{bmatrix},
\]

where \(\varphi_{i,t}\) denotes the shadow price of capital in firm \(i\) (the Lagrange multiplier associated with the capital accumulation constraint). This set of variables is minimal in the sense that knowledge of the coefficients \(A, B, C, D\) and \(G\) allows any agent in the economy to deduce and/or forecast the value of any variable of interest in the economy. That is, this is the minimal set of variables such that knowing their perceived law of motion is enough to recover the actual law of motion for all other variables in the model.
The first-order conditions determine a system of linear equations that describe the dynamic behavior of all variables in the model. Substituting the linear guess for the perceived law of motion of the forward-looking variables allow us to recover the actual law of motion. Then, from the first-order conditions, after some lengthy algebra one obtains

\[
\begin{bmatrix}
  c_t \\
  q_{i,t} \\
  \varphi_{i,t} \\
  x_{i,t}
\end{bmatrix} = A_{ALM} \theta_t + B_{ALM} \theta_{i,t} + C_{ALM} s_t + D_{ALM} e^{\text{noise}}_{i,t} + G_{ALM} \theta_t,
\]

where the coefficients \( A_{ALM}, B_{ALM}, C_{ALM}, D_{ALM} \) and \( G_{ALM} \) are nonlinear functions of the guess \( A, B, C, D \) and \( G \). The equilibrium is found by matching the coefficients, or equivalently, finding the fixed point of the operator that transforms the perceived law of motion into the actual law of motion. In general, there is no analytic solution, and numerical methods are required to find the fixed point.

### 3.7 Stock price as an endogenous signal

In this sub-section I study the equilibrium in the stock market, which is the novel addition to an otherwise standard RBC framework. The demand for the stock of firm \( i \) by trader \( j \) satisfies the first-order condition

\[
E_{j,i,t} [\Lambda_t (Q_{i,t} - D_{i,t} + \tau (B_{j,i,t} - 1) K_{i,t-1})] = E_{j,i,t} [\beta \Lambda_{t+1} Q_{i,t+1}].
\]

The trader must be indifferent between not buying the marginal share and transferring the marginal cost to the household (thus allowing the consumer to enjoy higher current consumption) and buying the marginal share and waiting for its future resale value. This Euler equation gives rise to the log-linearized demand

\[
b_{j,t} = \frac{1}{\tau \beta} ((1 - \beta) d_{i,t} - q_{i,t} + \beta E_{j,t} [\Lambda_{t+1} - \lambda_t + q_{i,t+1} + (1 - \sigma) \Delta a_t]),
\]

where lowercase variables denote log-linear deviations of stationary variables around the steady state, and where I drop the firm index \( i \) from \( b_{j,i,t} \) and \( E_{j,i,t} \) for convenience.

Trader \( j \) does not know the value of \( \lambda_t \), the marginal utility of consumption, because financial trade happens before consumption. She knows all the realization of past aggregate variables and the common beliefs about the permanent component of productivity, summarized in the vector of public information, \( S_t \). Additionally, she observes the current productivity, \( a_t \), her private signal \( s_{j,t} \), the stock price \( q_{i,t} \) and the current state of the firm.

To explicitly find the demand of trader \( j \) for the stock of the firm, we need to characterize
the information contained in the stock price. In the linear equilibrium,

\[ q_{i,t} = A_q \theta_t + B_q \theta_{i,t} + C_q s_t + D_q e_{i,t}^{\text{noise}} + G_q \theta_b, \]

for some coefficients \( A_q, B_q, C_q, D_q \) and \( G_q \). Since the vectors \( \theta_t, \theta_{i,t} \) and \( \theta_b \) are known, then the observation of the price is equivalent to the observation of the signal

\[ s_{i,t} \equiv q_{i,t} - A_q \theta_t - B_q \theta_{i,t} - G_q \theta_b = \alpha_a a_t + \alpha_s s_t + \alpha_e e_{i,t}^{\text{noise}}, \quad (1) \]

where \( C_q = \begin{bmatrix} \alpha_a & \alpha_s \end{bmatrix} \) and \( D_q = \alpha_e \). It follows that the expectation operator of trader \( j \) satisfies

\[ E_{j,t}[a_t, \theta_t, \theta_{i,t}, \theta_b, s_t, z_t] = E_{j,t}[a_t, \theta_t, \theta_{i,t}, \theta_b, s_t, z_t]. \]

All her signals are Gaussian-linear, and the signal extraction problem can be solved by standard Kalman filtering.

To confirm that the stock price can be expressed as the linear function we guessed, it is necessary to impose market clearing. The total supply of the share is constant and equal to one. Therefore it never deviates from its value of steady state. In the log-linear equilibrium, market clearing requires

\[ \int_0^1 b_{j,t} dj + e_{i,t}^{\text{noise}} = 0, \]

where total demand is just the sum of the aggregate demand by the informed traders and the demand by noise traders. Then, the equilibrium stock price satisfies

\[ q_{i,t} = (1 - \beta) d_{i,t} + \beta \int_0^1 E_{j,t}[(\lambda_{t+1} - \lambda_t + q_{i,t+1} + (1 - \sigma) \Delta a_t)] dj + \beta \tau e_{i,t}^{\text{noise}}. \]

In the log-linear equilibrium, \( \lambda_t, \lambda_{t+1}, q_{i,t+1} \) and \( d_{i,t} \) are linear functions of the observed vectors \( \theta_t, \theta_{i,t}, \theta_b \), the signals \( s_t \) and the shock \( e_{i,t}^{\text{noise}} \). Since the price signal is linear Gaussian, the expectation of the traders is linear as well. This confirms that the market-clearing price is linear and gives rise to the price signal \( s_{i,t} \).

### 3.7.1 The effect of information quality in the stock market

To disentangle the effects that changes on the quality of information have on the economy, I begin by describing how the information parameters affect the stock price. For ease of exposition, I consider a simpler information structure. The results obtained here, of course, extend to the general case.

For simplicity, assume that the aggregate signal received by the financial sector is noiseless: \( s_t^f = z_t \). In this case, the permanent component of productivity is learned by the consumer at
the end of each period, and the common belief held by all agents at the beginning of period \( t \) satisfies

\[ z_{t-1|t-1} = z_{t-1}. \]

Then the signal extraction problem reduces to a static problem. Note that the exogenous signals received by trader \( j \) can be expressed as

\[
a_t = E[z_t|z_{t-1}, z_{t-2}] + \rho_v v_{t-1} + \mu_t + \varepsilon_t,
\]

\[
s_{j,t} = E[z_t|z_{t-1}, z_{t-2}] + \varepsilon_t + \varepsilon_{j,t},
\]

where

\[ E[z_t|z_{t-1}, z_{t-2}] = (1 + \rho_z) z_{t-1} - \rho_z z_{t-2}. \]

Finally, the stock price gives origin to the signal (equation (1))

\[
s_{i,t} = \alpha a_t + \alpha_s z_t + \alpha_e e_{i,t}^{noise}
= \alpha a_t + \alpha_s E[z_t|z_{t-1}, z_{t-2}] + \alpha_s \varepsilon_t + \alpha_e e_{i,t}^{noise}.
\]

Note that \( v_{t-1} \) is known, since \( a_{t-1} \) and \( z_{t-1} \) have been observed at time \( t \). Let \( \tilde{a}_t = \mu_t + \varepsilon_t \), \( \tilde{s}_{j,t} = \varepsilon_t + \varepsilon_{j,t,} \), \( \tilde{s}_{i,t} = \varepsilon_t + \frac{\alpha_e}{\alpha_s} e_{i,t}^{noise} \), denote the component of the signals that is orthogonal to information known by the trader.

Then,

\[
E[z_t|a_t, s_{j,t}, s_{i,t}] = E[z_t|z_{t-1}, z_{t-2}] + E[\varepsilon_t|a_t, s_{j,t}, s_{i,t}]
= E[z_t|z_{t-1}, z_{t-2}] + E[\varepsilon_t|\tilde{a}_t, \tilde{s}_{j,t}, \tilde{s}_{i,t}].
\]

Finally, from the algebra of the conditional expectation of jointly normal random variables, it follows that

\[
E[\varepsilon_t|\tilde{a}_t, \tilde{s}_{j,t}, \tilde{s}_{i,t}] = \frac{\tau_\mu \tilde{a}_t + \tau_{disp} \tilde{s}_{j,t} + \left(\frac{\alpha_s}{\alpha_e}\right)^2 \tau_{noise} \tilde{s}_{i,t}}{\tau_\mu + \tau_{disp} + \left(\frac{\alpha_s}{\alpha_e}\right)^2 \tau_{noise}},
\]

where \( \tau_k = \sigma_k^{-2}, \ k \in \{\mu, noise, disp\} \), denotes the precision (inverse variance) of the \( k \)-th innovation.

Consider now the effect of having less dispersed information in the financial sector, summarized by an increase in \( \tau_{disp} \). Keeping for now the price signal as exogenous, less dispersed information is perceived by each trader as an increase in the precision of her private signal. Given the solution to the signal extraction problem, it is optimal to assign a larger relative weight to the private information when forming expectations about the permanent shock to productivity. A similar effect holds if there is a reduction in the volatility of the noisy trade. In that case, for a given ratio \( \left(\frac{\alpha_e}{\alpha_s}\right) \), the stock price is perceived as a more precise signal, and
it is optimal to assign a larger relative weight to it when forming expectations.

Of course, these are partial equilibrium effects. The stock price is determined by the aggregate expectation of traders, and those expectations change with changes in the precision of the signals they observe. For example, when traders assign a relatively larger weight to their private information and a relatively lower weight to the stock price, their expectations reflect more their private information (i.e., innovations to private signals become more important in explaining the variability in the conditional expectations). Then, aggregate expectations depend relatively more on the aggregate signal received by the financial sector, and relatively less in the stock price itself and other shocks. In equilibrium, the stock price satisfies market clearing and it reflects the aggregate expectations. Thus, the absolute coefficient \( \left| \frac{\alpha_e}{\alpha_s} \right| \) decreases: the price signal becomes relatively more precise, because it aggregates the more precise private information.

This is the microeconomic mechanism at the core of the noisy rational expectations paradigm. If the financial sector becomes better at acquiring information, if information is less dispersed among financial intermediaries or if there is less noisy trade, then the stock price transmits more precise information about future productivity. Since the real sector observes stock prices and base production and investment decisions on this information, then the macroeconomic aggregates, in principle, respond more to the permanent component of productivity. How good are stock prices at transmitting information and how useful is this information for the real economy are the quantitative questions that will be assessed in the next section.

4 Quantitative analysis

In this section I perform a quantitative analysis of the social value of the information about future productivity that is aggregated and transmitted through the financial system.

4.1 Calibration

Following the classic literature on real business cycles, I calibrate the model to a quarterly frequency. Given the choice of period length, I set the discount factor \( \beta \) equal to 0.99 and the depreciation rate \( \delta \) equal to 0.025. The production parameter \( \alpha \) is set to the standard value of \( \frac{1}{3} \). For the preference parameters, I set the coefficient of risk aversion, \( \sigma \) equal to 2, and the inverse of the Frisch elasticity of labor supply, \( \eta \) equal to one.

The main driver of economic fluctuations in the model is aggregate productivity. The behavior of the log-growth rate of productivity is governed by four parameters: the persistence of the permanent component \( \rho_z \), the variance of the permanent shock \( \sigma_z^2 \), the persistence of the transitory component \( \rho_v \), and the variance of the transitory shock, \( \sigma_v^2 \). I use the quarterly, utilization-adjusted series on total factor productivity constructed by Fernald [2012] as the
data equivalent to the model’s aggregate productivity. I choose the parameters to match the variance, first-order, second-order and third-order autocorrelation coefficients from the observed log-growth rate.

Next I turn to the firm-specific component of productivity. Using annual data on firm-level characteristics, Imrohoroglu and Tuzel [2014] estimate the idiosyncratic component of firm-level TFP. They find the persistence of annual firm-level TFP to be 0.703 and the standard deviation of the TFP shock to be 0.268. These estimates imply quarterly values of $\rho_\xi = 0.9157$ and $\sigma_\xi = 0.1515$.

The model depends on 21 parameters, 11 of which have been calibrated so far. The remaining 10 parameters include the persistence and volatility of the capital embedded shock ($\rho_\chi$, $\sigma_\chi^2$), the persistence and volatility of the marginal utility shock ($\rho_\vartheta$, $\sigma_\vartheta^2$), the habit formation parameter $h$, the scale of the investment adjustment cost $\phi$ and the elasticity parameter on the utilization cost $\varsigma$. These parameters determine the volatility and correlation properties of aggregate time series like the growth rate of consumption, investment and the stock market index. Therefore, the standard practice is to calibrate these parameters to target some relative variances and autocorrelations from aggregate time series in the data.

The last three parameters are the information parameters, i.e., the variance of the noisy trade, $\sigma_{\text{noise}}^2$, the dispersion of the signals received by the financial traders, $\sigma_{\text{disp}}^2$, and the variance of the noise to the aggregate financial signal, $\sigma_f^2$. The information parameters govern the equilibrium expectations of firms, workers and financial traders. In particular, they determine how informative the stock price is for investment, how strong is the relation between the stock price and the permanent component of productivity, and how disperse are the beliefs of the financial traders.

For the identification strategy, I follow David et al. [2013]. In the model, the stock price is informative for investment. If the uncertainty of the firms is high, they rely heavily on stock prices to make inferences about aggregate and future conditions, increasing the correlation between investment growth and stock market returns. However, a large correlation could also arise if firms are well informed, because then both the stock market and investment follow the same fundamentals closely. Therefore, it is necessary to take into account the relation between stock prices and the permanent component of productivity. To do so, I consider the correlation between stock returns and lagged productivity growth. In the data, I use the S&P500 stock market index as the aggregate measure of stock prices, and the quarterly, utilization-adjusted TFP series built by Fernald [2012] as the measure of aggregate productivity.

Finally, I take into account the effect of the information parameters in the dispersion of beliefs among traders. From the Survey of Professional Forecasters (Philadelphia FED) I recover measures on cross-sectional dispersion of beliefs about the growth rate of consumption, GDP, profits of corporations and nonresidential investment. I assume that the dispersion of beliefs of the financial traders corresponds to the dispersion measured in the data. As in Hassan and
Table 2: Baseline calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preferences/production</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Production function</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1.00</td>
<td>Inv. Frisch elasticity</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.00</td>
<td>Risk aversion</td>
</tr>
<tr>
<td>Aggregate productivity process</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.7414</td>
<td>Permanent comp.</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.0024</td>
<td></td>
</tr>
<tr>
<td>$\rho_v$</td>
<td>0.4526</td>
<td>Transitory comp.</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>0.0065</td>
<td></td>
</tr>
<tr>
<td>Firm-specific productivity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_\xi$</td>
<td>0.9147</td>
<td>Imrohoroglu and Tuzel [2014]</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>0.1517</td>
<td></td>
</tr>
<tr>
<td><strong>Estimated parameters (SMM)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>0.3160</td>
<td>Habit</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.6275</td>
<td>Adjustment cost</td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>1.8711 $\times$ $10^4$</td>
<td>Utilization cost</td>
</tr>
<tr>
<td>Capital-embodied shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_X$</td>
<td>0.7072</td>
<td></td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>0.0112</td>
<td></td>
</tr>
<tr>
<td>Marginal-utility shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_\vartheta$</td>
<td>0.9898</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\vartheta$</td>
<td>0.0016</td>
<td></td>
</tr>
<tr>
<td>Information parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>0.0042</td>
<td>Aggregate signal</td>
</tr>
<tr>
<td>$\sigma_{disp}$</td>
<td>0.0193</td>
<td>Dispersion</td>
</tr>
<tr>
<td>$\sigma_{noise}$</td>
<td>0.00449</td>
<td>Noisy trade</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.0996</td>
<td>Portfolio adj. cost</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hassan and Mertens [2014]</td>
</tr>
</tbody>
</table>

Quarterly calibration.
Mertens [2014], the model fails to generate the level of observed dispersion of beliefs under reasonable parameterizations. Therefore, I focus on ratios of dispersion of beliefs. Specifically, I consider the ratio of the dispersion of profits growth to the dispersion of investment growth, $\sigma_j(E_{j,t}g_{dt+1})/\sigma_j(E_{j,t}g_{xt+1})$, and the ratio of the dispersion of growth of consumption to the dispersion of growth of output, $\sigma_j(E_{j,t}g_{ct+1})/\sigma_j(E_{j,t}g_{yt+1})$. Figure 6 illustrates how these moments respond to selected information parameters, under the baseline calibration.

Figure 6: Selected moments - Identification

To pin down the value of the parameters, I use a simulated method of moments (SMM) approach. I estimate the parameters to minimize the loss function

$$(M(\theta) - m)' W (M(\theta) - m),$$

where $M(\theta)$ is the vector of moments generated by the model under the parameter vector $\theta$, $m$ is the vector of data targets for those moments, and $W$ is a diagonal weighting matrix. The $k$-th entry in the diagonal of $W$ is $\frac{1}{m_k^2}$, where $m_k$ is the $k$-th entry in the data target vector $m$.

The first panel of Table 3 presents the moments used in the SMM estimation, the data targets and the simulated moments generated by the model. Under the baseline calibration, the model does a reasonable job fitting the correlations and autocorrelations of the growth rate of consumption, investment and the stock market index. The model fails in fitting the time series volatility of the growth rate of the stock price and the cross-sectional dispersion of stock returns, and underestimates the ratio of dispersion of beliefs $\sigma_j(E_{j,t}g_{dt+1})/\sigma_j(E_{j,t}g_{xt+1})$. 

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Table 3: Macroeconomic moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Baseline</th>
<th>No dispersed</th>
<th>No noise</th>
<th>No fin. shock</th>
<th>Common</th>
<th>Full info</th>
<th>No info</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma (gy)$ (%)</td>
<td>0.53</td>
<td>0.57</td>
<td>0.57</td>
<td>0.57</td>
<td>0.56</td>
<td>0.50</td>
<td>0.54</td>
<td>0.45</td>
</tr>
<tr>
<td>$\sigma (gc)$ (%)</td>
<td>0.39</td>
<td>0.45</td>
<td>0.46</td>
<td>0.46</td>
<td>0.41</td>
<td>0.41</td>
<td>0.39</td>
<td>0.44</td>
</tr>
<tr>
<td>$\sigma (gx)$ (%)</td>
<td>2.03</td>
<td>1.62</td>
<td>1.61</td>
<td>1.61</td>
<td>1.80</td>
<td>1.70</td>
<td>1.99</td>
<td>1.27</td>
</tr>
<tr>
<td>$\sigma (gq)$ (%)</td>
<td>7.78</td>
<td>0.78</td>
<td>0.76</td>
<td>0.76</td>
<td>0.90</td>
<td>0.84</td>
<td>0.98</td>
<td>0.56</td>
</tr>
<tr>
<td>$\rho (gq, gx)$</td>
<td>0.0210</td>
<td>0.0170</td>
<td>-0.0061</td>
<td>-0.0072</td>
<td>0.1765</td>
<td>0.0802</td>
<td>0.274</td>
<td>-0.635</td>
</tr>
<tr>
<td>$\rho (gc, gx)$</td>
<td>0.233</td>
<td>0.203</td>
<td>0.193</td>
<td>0.192</td>
<td>0.101</td>
<td>-0.057</td>
<td>-0.072</td>
<td>-0.037</td>
</tr>
<tr>
<td>$\rho (gq, \Delta a_{t-1})$</td>
<td>-0.086</td>
<td>-0.059</td>
<td>-0.0079</td>
<td>-0.0056</td>
<td>0.057</td>
<td>0.262</td>
<td>0.181</td>
<td>0.477</td>
</tr>
<tr>
<td>$\sigma_j(E_j,t_{gt+1})/\sigma_j(E_j,t_{gx+1})$</td>
<td>3.305</td>
<td>2.089</td>
<td>0.909</td>
<td>1.8946</td>
<td>2.4478</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_j(E_j,t_{gt+1})/\sigma_j(E_j,t_{gy+1})$</td>
<td>0.972</td>
<td>1.032</td>
<td>0.639</td>
<td>0.337</td>
<td>7.299</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho (gc)$</td>
<td>0.313</td>
<td>0.253</td>
<td>0.215</td>
<td>0.216</td>
<td>0.411</td>
<td>0.433</td>
<td>0.464</td>
<td>0.396</td>
</tr>
<tr>
<td>$\rho (gx)$</td>
<td>0.306</td>
<td>0.257</td>
<td>0.264</td>
<td>0.264</td>
<td>0.286</td>
<td>0.323</td>
<td>0.328</td>
<td>0.344</td>
</tr>
<tr>
<td>$\rho (gq)$</td>
<td>-0.0404</td>
<td>-0.0467</td>
<td>-0.0256</td>
<td>-0.0254</td>
<td>-0.0719</td>
<td>-0.0598</td>
<td>-0.0635</td>
<td>-0.0086</td>
</tr>
<tr>
<td>$\sigma_i (gq_i)$ (%)</td>
<td>7.27</td>
<td>3.60</td>
<td>3.59</td>
<td>3.59</td>
<td>3.61</td>
<td>3.59</td>
<td>3.59</td>
<td>3.59</td>
</tr>
</tbody>
</table>

No dispersed information: $\sigma_{disp} = 0$; No noise traders: $\sigma_{noise} = 0$; No financial shock: $\sigma_f = 0$.
Common information: the aggregate signals $a_t$ and $s^f_t$ are observed by all agents.
Full information: common information, and $s^f_t = z_t$, the permanent component productivity is commonly observed.
No information: common information, no signals about $z_t$.
4.2 Dynamic effect of imperfect information on the economy

Information is acquired and aggregated by the financial system, and transmitted to the real economy through asset prices. As a first step to understand how this information affects the real economy, we can analyze how some statistical properties of aggregate macroeconomic variables change when the parameters controlling the informativeness of the stock price change. First, I illustrate how the presence of imperfect and dispersed information affects the dynamic behavior of the economy. In particular, I compare the impulse response function to a temporary productivity shock under imperfect information with the response under full information. Then, I study how selected variances and covariances of macroeconomic aggregates respond to changes in the dispersion of information among traders, the volatility of the noisy trade and the overall quality of the information aggregated by the financial system.

4.2.1 Impulse response

Figure 7 presents the dynamic response of consumption, investment, output and the aggregate stock price to a innovation to the temporary component of productivity.

![Figure 7: Impulse response temporary shock to productivity](image)

Solid line: baseline economy.
Dashed line: full information (all agents know both the level of productivity $a_t$ and the level of the permanent component $z_t$).

At time zero the economy is in the deterministic steady state. The shock occurs once at period one and its size is equal to one standard deviation (0.065). The shock causes a temporary increase of aggregate productivity above its steady-state level and acts as a positive shock to aggregate supply. Under imperfect information, the consumer cannot distinguish temporary
from permanent shocks, and believes that the observed increase in aggregate productivity is
due in part to a permanent shock. Optimally, consumption increases by more than it would if
the consumer knew for sure that the shock is transitory. The response of output both under
imperfect and full information is similar, due to the fact that capital is predetermined. The
aggregate resource constraint then requires investment to increase by less under imperfect
information. The response of the stock price is dampened under imperfect information as
well. From the impulse response function, it can be concluded that the presence of imperfect
information dampens the dynamic response of investment and asset prices to productivity
shocks, and this effect is significant. Thus, information frictions have an important effect on
the dynamic behavior of the economy.

4.2.2 Macroeconomic moments

I study how the statistical moments of key macroeconomic aggregates respond to changes in
the informational features of the financial system.

First, I consider the dispersion of information. I compare the baseline economy to an
economy in which traders observe the aggregate financial signal without idiosyncratic noise.
Most of the statistical moments considered show little change. The correlation between the
growth rate of investment and the growth rate of the stock price index decreases slightly.
The reason is that the stock price is more informative about the permanent component of
productivity. Thus, the price depends more on the aggregate financial signal, and less on
other shocks, like noisy trade, aggregate productivity or common priors. At the same time,
firms put a larger weight on the price signal when making decisions, but keep a relatively
constant response to their own productivity and to the priors. The net effect of this is a lower
correlation between investment growth and stock returns. Finally, the relative disagreement
about the growth rate of dividends and consumption decreases. Similar results hold for the
variance of the noisy trade.

Consider next the overall quality of the information available to the financial sector. The
aggregate financial signal is observed by the consumer as well, and its effect on macroeconomic
aggregates is larger. When the consumer observes the permanent component of productivity,
the prior common beliefs become more informative about the future as well. Firms rely more
heavily on the asset price and on the priors when forming expectations. This increases the
correlation between investment and the stock price index. On the other hand, it decreases
the correlation of investment and consumption growth, which depends relatively more on
the current realization of the permanent component of productivity. As such, consumption
becomes more persistent and its volatility decreases. At the same time, the volatility of the
growth rate of aggregate investment increases, because firms respond more strongly to the
stock price signal and its noise. Finally, lagged productivity growth is better at forecasting
stock market return, because it depends more on the permanent component of productivity.
Lastly, I analyze the case when information is common across all agents: current aggregate productivity \(a_t\), and the aggregate financial signal \(s^f_t\), are observed by all agents at the beginning of the period, and the financial system plays no role in aggregating or transmitting information. This corresponds closely to the benchmark established in the representative-agent economy studied in Section 2. The third panel of Table 3 summarizes the results. Common information reduces the volatility of the growth rate of consumption and output, because the noise of the financial system does not affect real decisions anymore. Traders and firms observe consumption, and the signal about the permanent component of productivity behaves as a news shock. It is well known that in the basic RBC framework, positive news shocks tend to generate a boom in consumption and a bust in investment (Jaimovich and Rebelo [2009]). Under common information, the comovement problem becomes more relevant, and as result the correlation between consumption growth and investment growth falls.

The precision of the aggregate financial signal is the only information parameter that affects the economy when information is common. The full information case corresponds to a completely precise signal \((s^f_t = z_t)\), such that the permanent component of productivity is observed. Investment is more sensible to the permanent component, and the variance of its growth rate increases. On the other hand, when the aggregate financial signal is completely noisy, the economy receives no information about \(z_t\). In the no information case, the variance of investment is lower. Other moments do not show significant changes.

To summarize, the information parameters that characterize the information features of the financial system in the model economy have very small effects on the statistical properties of macroeconomic aggregates. The quality of the aggregate financial signal has a larger effect because the consumer directly observe this signal, but noisy trade and dispersion do not have a strong impact on the economy.

4.3 Welfare

In this subsection I study the effect of the information aggregated and transmitted through the financial system on social welfare. As in Section 2, the welfare criterion is given by the expected value, at time zero, of the discounted sum of the utility expected by the consumer, under the assumption that the economy starts at the deterministic steady state. To determine the social value of information, I compare the value of the welfare criterion in the economy under the baseline calibration, with the welfare value in alternative economies with different information parameters. To interpret the changes in welfare, I consider the permanent stationary consumption subsidy/tax required to keep the household indifferent between facing the baseline economy or living in the alternative economies.

Table 4 presents the permanent compensation in annual consumption required to induce the consumer to stay in the baseline economy. A positive compensation means that the consumer is better off in the alternative economy and must be paid off to stay in the baseline
Table 4: Annualized consumption compensation (%)

<table>
<thead>
<tr>
<th>No dispersed</th>
<th>No noise</th>
<th>No fin. shock</th>
<th>Dispersed</th>
<th>No signal</th>
<th>Common</th>
<th>Full info</th>
<th>No info</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0462</td>
<td>-0.0009</td>
<td>-0.0633</td>
<td>0.0045</td>
<td>0.0529</td>
<td>-0.0631</td>
</tr>
</tbody>
</table>

No dispersed information: \( \sigma_{\text{disp}} = 0 \); No noise traders: \( \sigma_{\text{noise}} = 0 \); No financial shock: \( \sigma_f = 0 \).

Dispersed: \( \sigma_{\text{disp}} \to \infty \); No signal: \( \sigma_f \to \infty \).

Common information: the aggregate signals \( a_t \) and \( s^f_t \) are observed by all agents.

Full information: common information, and \( s^f_t = z_t \), the permanent component productivity is commonly observed.

No information: common information, no signals about \( z_t \).

A negative compensation implies that the consumer is willing to give up some consumption to move from the alternative economy to the baseline economy. That is, welfare under the alternative is lower.

First, I compare the baseline economy with the case of no dispersed information: financial traders observe the aggregate signal without noise. In this case, the stock price is more informative about the permanent component of productivity. Firms decide on investment and production with better information. This generates a negligible welfare gain: the consumer would be willing to pay 0.0002% of her consumption to move to the economy without dispersed information.

A very similar result holds for the noisy trade. In the economy without noisy trade, the stock price becomes a more precise signal about productivity. Firms and workers are better informed, but the effect on welfare is negligible. The consumer would be willing to pay 0.0002% of her annual consumption to move to the economy without noisy trade. A household consuming on average $50,000 a year would be willing to pay less than ten cents to eliminate noisy trade.

Then, I consider the effect of the overall quality of the aggregate signal received by the financial sector. More precise financial information increases the information contained in the stock price. But it also enhances the ability of the consumer to forecast the permanent component of productivity, and improves the quality of the common priors held at the beginning of each period. Therefore, production, investment and consumption decisions are made with better information, and this generates a modest but significant increase in welfare. The consumer would pay up to 0.0462% of her annual consumption for the aggregate financial signal to be noiseless.

Finally, I compare the economy under common, full information, where aggregate productivity and its separate components are observed by all the agents, with the economy under which there is no information at all about the permanent component but aggregate productivity is commonly known. The consumer must receive a permanent increase of 0.116% on
annual consumption to be willing to give up her knowledge about \( z_t \). These results are in line with the analysis performed in the representative-agent economy in Section 2: for the same degree of persistence of the permanent component of productivity, the representative agent required a compensation of 0.088%.

The small change that the information parameters induce on welfare could be due to the fact that, under the baseline calibration, moving to the alternative economies implies only a small change in the relevant information parameters. To discount for this possibility, it is necessary to study the response of welfare to large changes in the information parameters. Figure 8 presents the permanent compensation in annual consumption as a function of the different information parameters. The required compensation is decreasing in the variance of the noisy trade and in the degree of information dispersion. As before, for reasonable levels of the information parameters, the consumption compensation is very small and almost negligible. The compensation is decreasing in the variance of the noise of the aggregate signal, and it takes modest but more significant values.

![Figure 8: Annualized consumption compensation (%)](image)

To get some intuition for these results, let us go back to the signal extraction problem of the financial traders. As before, in equilibrium, the observation of the stock price is equivalent, for the traders, to the observation of the price signal

\[
\tilde{s}_{i,t} = z_t + \alpha_f e_{f,t} + \frac{\alpha_e}{\alpha_s} e_{\text{noise},t}.
\]

The variance of the noise component of the signal is

\[
\sigma_{\tilde{s}|z}^2 = \sigma_f^2 + \left( \frac{\alpha_e}{\alpha_s} \right)^2 \sigma_{\text{noise}}^2.
\]
and it measures how informative the stock price is about the permanent component.

Consider a reduction in the variance of the aggregate financial shock, $\sigma_f^2$. This has a direct effect on $\sigma_{f|z}^2$, making the price signal more informative, and an indirect effect, through general equilibrium. As discussed in Section 3.7.1, $\left(\frac{\sigma_f}{\sigma_s}\right)^2$ decreases with $\sigma_f^2$, thus further increasing the precision of the market signal. Then, the direct and indirect effect on price informativeness of a reduction in $\sigma_f^2$ complement each other. This feature, together with the fact that the consumer directly observes the aggregate financial signal, implies that the effect of $\sigma_f^2$ on the economy can be large.

On the other hand, a reduction on the dispersion of the information held by financial traders has no direct effect on the precision of the market signal. The indirect effect, mediated through general equilibrium, reduces the variance of the noise of the price signal, but its impact is weaker than in the case of the aggregate financial shock. Finally, a reduction in the variance of the noisy trade, $\sigma_{\text{noise}}^2$, increases the precision of the price signal, but it also induces an increase in the coefficient $\left(\frac{\sigma_f}{\sigma_s}\right)^2$, dampening its effect on the informativeness of the stock price. Thus, the logic of the general equilibrium indicates that the reduction in dispersion or in noisy trade has a small effect on the real economy.

### 4.4 The informativeness of financial markets

The small effects on the economy caused by changes in the information parameters could be due to two reasons: first, firms learn a substantial amount from the stock price, but in equilibrium the information they extract is not useful at all; or second, the information structure and the constraints imposed by the calibration are such that, even if the stock price potentially contains a lot of information, firms do not learn much from it. To measure the amount of learning, I consider the Root Mean Squared Error (RMSE): on average, how far are the forecasts of the agents in the economy from the actual fundamentals.

I study the forecast of the permanent component of productivity, $z_t$. The permanent component of productivity is not stationary, and its unconditional variance is not defined. Therefore, I normalize the RMSE of $z_t$ by the RMSE that would be obtained if the consumer observed only aggregate productivity, $a_t$, and no other signal, to forecast $z_t$. Numbers close to one imply that there is little learning. Figure 9 shows the results. Less information is detrimental to the quality of the forecast of traders and consumers. However, changes in the volatility of the noisy trade or in the dispersion of information have no significant effect on the ability of producers to forecast the permanent component of productivity. The reason is that, in equilibrium, the stock price signal depends on both the current aggregate productivity, $a_t$, and on the signal about its permanent component, $s^f_t$. Both are informative about the permanent component of productivity, and changes in the information parameters change the relative response of the stock price to $a_t$ and $s^f_t$. In equilibrium, these relative changes tend
to offset the effect of noise and dispersion, keeping the real informativeness of the stock price more or less constant. To see this, substitute the process for productivity and the financial signal into the stock market price signal, as defined in equation (1):

$$s_{i,t} = \frac{\alpha_a a_t + \alpha_s s_{f} + \alpha_e e_t^{\text{noise}}}{\alpha_s} = \left(1 + \frac{\alpha_a}{\alpha_s}\right) z_t + \frac{\alpha_a}{\alpha_s} v_t + e_t^{f} + \frac{\alpha_e}{\alpha_s} e_t^{\text{noise}}.$$

A reduction in the variance of the noisy trade, or a decrease in the dispersion of information, generate a decrease in $|\alpha_a/\alpha_s|$. In principle, the stock price becomes more informative, because it depends less on noise shocks. However, there is also a decrease in $|\alpha_a/\alpha_s|$. The stock price responds less strongly to aggregate productivity, because investment and consumption rely more on $s_{f}$. Depending on the equilibrium value taken by the price signal coefficients, the reduction in $\alpha_a$ can offset the reduction in $|\alpha_a/\alpha_s|$, keeping the informativeness of $s_{i,t}$ about $z_t$ the same.

This result depends on the fact that firms do not observe aggregate productivity, but the stock price depends on it. The informational features of the financial system have very small effects on the economy because, in equilibrium, there are no changes in the ability of firms to learn from stock prices. It can be argued, however, that if firms observe the aggregate productivity, then they are able to extract all the information contained in the stock price about future productivity. A stronger learning then could generate important effects on welfare and macroeconomic moments.

To test for this possibility, I consider an alternative information structure, in which all
agents in the economy observe the level of aggregate productivity, $a_t$. The stock price becomes a signal only about the permanent component of productivity, noised up by random trade. I redo the calibration of the model under the new information structure, using the same Simulated Method of Moments (SMM) procedure. Under the new calibration, the economy moves towards less aggregate information. The variance of the noisy trade is larger, and signals are less informative. It is important to note that the fit of the model is significantly worse under the alternative information structure.

Table 5 in Appendix B presents the results for welfare. All the qualitative results hold. Under the alternative information structure, changes in noisy trade and in the dispersion of information have negligible effects on welfare. The reason is that under the alternative calibration the noisy trade is relatively large, making the stock price not informative. On the other hand, there are larger welfare gains when increasing the overall quality of the information aggregated by the financial markets. The reason is that the economy, under the alternative calibration, is closer to the no information case.

5 Conclusions

In this paper I studied the value of information about future productivity in a RBC framework, and analyzed the ability of the financial system to transmit this information to the real economy. I established that information about the long-run value of productivity is valuable for a central planner and it enhances welfare. Although the allocation of resources over time improves with more information, most of the welfare gain is explained instead by the reduction on risk associated with having more information about the long-run. This is specially true for recursive utility preferences. Standard calibrations used in the macro-finance literature assign a large value to the coefficient of risk aversion and imply that a central planner endowed with a Epstein-Zin recursive utility function has a strong preference for early resolution of uncertainty. The observation of any information regarding the long-run value of productivity helps the planner to partially resolve long-run uncertainty, thus generating large welfare gains, even if the allocation of resources does not change in any significant way.

Then, I included a simple financial sector in the RBC framework as the standard noisy rational expectations stock market. The stock price aggregates and transmits information about current and future productivity, and the weight it assigns to each component is endogenously determined in equilibrium. Changes in the informational features of the stock market are reflected in the information content of the stock price. Since the price is used as a signal by firms, it affects how production, investment and dividends respond to information. Because the stock market reflects expectations on dividends, changes in the real economy feedback to the stock price and affect its informativeness.

The feedback mechanism between information parameters, stock price and the real econ-
omy are at the heart of the quantitative results of the paper, and it has not been studied in a macroeconomic framework before. I showed that changes in the informational features of the financial system that keep the precision of the overall information available to the economy constant have very little effects on macroeconomic variables or on welfare. The reason is that, in equilibrium, the information content of the stock price shifts between current and future productivity, keeping the informativeness about long-run productivity more or less constant. Firms do not improve their ability to learn from the stock prices, and because the overall information is constant, there are very little effects on the real economy.

Finally, I showed that increasing the precision of the aggregate information available to the financial system has a positive but modest effect on welfare. The immediate policy implication is that regulators should aim to an increase in the information acquired by the financial system as a whole, and not on interventions that reduce how dispersed information is among agents. The point to take away, however, is that the effect of the information transmitted through asset prices to the real economy is relatively small in this framework.

References


Joel David, Hugo A. Hopenhayn, and Vaidyanathan Venkateswaran. The Informativeness of


A Appendix: The representative-agent economy

A.1 CRRA equations

The solution to the household’s problem is characterized by the usual first-order conditions:

\[ C_t : \quad U'(C_t) = \Lambda_t, \]

\[ N_t : \quad A_t^{-\sigma} V'(N_t) = \Lambda_t \frac{(1-\alpha)Y_t}{N_t}, \]

\[ X_t : \quad \Lambda_t = \Phi_t \left( 1 - \phi \left( \frac{X_t}{X_{t-1}} - 1 \right) \right) + \beta E_t \Phi_{t+1} \left( \phi \left( \frac{X_{t+1}}{X_t} - 1 \right) \frac{X_{t+1}}{X_t} - \frac{\phi}{2} \left( \frac{X_{t+1}}{X_t} - 1 \right)^2 \right), \]

\[ K_t : \quad \Phi_t = \beta E_t \left( \Lambda_{t+1} \frac{\alpha Y_{t+1}}{K_t} + \Phi_{t+1} (1 - \delta) \right). \]

The productivity process \( A_t \) is not stationary, and aggregate variables must be normalized by the level of productivity. Let \( \hat{C}_t = \frac{C_t}{A_t}, \hat{X}_t = \frac{X_t}{A_t}, \hat{Y}_t = \frac{Y_t}{A_t}, \hat{K}_t = \frac{K_t}{A_t}, \hat{\Lambda}_t = \frac{\Lambda_t}{A_t^{\sigma-1}} \) and \( \hat{\Phi}_t = \frac{\Phi_t}{A_t^{\sigma-1}} \). Then, the optimality conditions reduce to

\[ \hat{\Lambda}_t = \hat{C}_t^{-\sigma}, \]

\[ \psi N_t^\alpha = \hat{\Lambda}_t \frac{(1-\alpha)\hat{Y}_t}{N_t}, \]

\[ \hat{\Lambda}_t = \hat{\Phi}_t \left( 1 - \phi \left( \frac{\hat{X}_t}{\hat{X}_{t-1}} \frac{A_t}{A_{t-1}} - 1 \right) \right) + \beta \left( \frac{A_t}{A_{t-1}} \right)^{-\sigma} E_t \Phi_{t+1} \left( \phi \left( \frac{\hat{X}_{t+1}}{\hat{X}_t} \frac{A_t}{A_{t-1}} - 1 \right) \frac{\hat{X}_{t+1}}{\hat{X}_t} \frac{A_t}{A_{t-1}} - \frac{\phi}{2} \left( \frac{\hat{X}_{t+1}}{\hat{X}_t} \frac{A_t}{A_{t-1}} - 1 \right)^2 \right), \]

\[ \hat{\Phi}_t = \beta \left( \frac{A_t}{A_{t-1}} \right)^{-\sigma} E_t \left( \hat{\Lambda}_{t+1} \frac{\alpha \hat{Y}_{t+1}}{K_t} + \hat{\Phi}_{t+1} (1 - \delta) \right), \]

and they characterize the optimal allocation, together with the constraints

\[ \hat{Y}_t = \hat{K}_t^\alpha \left( N_t \frac{A_t}{A_{t-1}} \right)^{1-\alpha}, \]

\[ \hat{K}_t \frac{A_t}{A_{t-1}} = (1 - \delta) \hat{K}_{t-1} + \hat{X}_t \left( 1 - \frac{\phi}{2} \left( \frac{\hat{X}_t}{\hat{X}_{t-1}} \frac{A_{t-1}}{A_{t-2}} - 1 \right)^2 \frac{\hat{X}_{t-1}}{\hat{X}_t} \frac{A_{t-1}}{A_{t-2}} \right). \]

The welfare value function satisfies the recursion

\[ V_t = U(C_t) - A_{t-1}^{-\sigma} V(N_t) + \beta E_t V_{t+1}, \]
and the stationary value is

\[
\hat{V}_t = \frac{V_t}{A_{t-1}^{1-\sigma}} = U \left( \hat{C}_t \right) - V (N_t) + \beta \left( \frac{A_t}{A_{t-1}} \right)^{1-\sigma} E_t \hat{V}_{t+1}.
\]

I compare the stationary value attained under full information at time zero, \(\hat{V}_{full} = E_0 \left[ \hat{V}_1 \left( I^{full} \right) \right]\), with the value attained under partial information, \(\hat{V}_{partial} = E_0 \left[ \hat{V}_1 \left( I \right) \right]\). To translate the welfare differences into meaningful units, define the certainty equivalent level of consumption in steady state,

\[
\hat{V}_k = U (C_k) - V (N),
\]

where \(k \in \{full, partial\}\) and \(N\) is the steady-state value of the labor hours. Consider the permanent subsidy/tax \(\tau\) required to keep the household indifferent between the full information economy and the partial information one:

\[
U \left( (1 + \tau) C_{partial} \right) - V (N) = \hat{V}_{full}.
\]

Since \(U (\cdot)\) is invertible, it follows that

\[
1 + \tau = \frac{C_{full}}{C_{partial}}.
\]

### A.2 Epstein-Zin

The aggregate endogenous states of the economy are \(X_t\) and \(K_t\). The utility function is defined recursively through the optimal value function \(V_t\), and in equilibrium \(V_t (X_{t-1}, K_{t-1})\) is a function of the endogenous state. Substituting the capital accumulation equation in \(V_{t+1}\), it can be shown that the efficient allocation is characterized by the first-order conditions

\[
N_t : \quad \frac{1 - o}{o} \cdot \frac{C_t}{1 - N_t} = (1 - \alpha) \frac{Y_t}{N_t}
\]

\[
X_t : \quad -V_t^{\psi} \hat{C}_t^{\psi} (1 - \beta) \left. \frac{\hat{C}_t}{C_t} \right|_{\gamma} + V_t^{\psi} \beta E_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{1-x}{1-\gamma}} E_t \left[ V_{t+1}^{1-\gamma} \right] \left( 1 - \phi \left( \frac{X_t}{X_{t-1}} - 1 \right) \right) = 0
\]

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and the envelope conditions

\[ V_{K,t} = V_t^{1-\Psi} \left( 1 - \beta \right)^\alpha \frac{C_t}{C_t^{1-\Psi}} \frac{Y_t}{K_{t-1}} + \]

\[ + V_t^{1-\Psi} \beta E_t \left[ V_{t+1}^{1-\gamma} \right]^{1-\Psi} E_t \left[ V_{t+1}^{1-\gamma} \times V_{K,t+1} \right] (1 - \delta) \]

\[ V_{X,t} = V_t^{1-\Psi} \beta E_t \left[ V_{t+1}^{1-\gamma} \right]^{1-\Psi} E_t \left[ V_{t+1}^{1-\gamma} \times V_{K,t+1} \right] \left( \phi \left( \frac{X_t}{X_{t-1}} - 1 \right) \frac{X_t}{X_{t-1}} - \frac{\phi}{2} \left( \frac{X_t}{X_{t-1}} - 1 \right)^2 \right), \]

where \( \tilde{C}_t \) denotes the Cobb-Douglas consumption-leisure composite,

\[ \tilde{C}_t = C_t^\alpha (A_t - 1)^{(1 - \gamma)}. \]

As in the CRRA case, the equations can be re-expressed in stationary terms. The normalized value function satisfies the recursion

\[ \hat{V}_t = \frac{V_t}{A_{t-1}} = \left( 1 - \beta \right) \left( \frac{C_t}{A_{t-1}} \right)^{1-\Psi} + \beta \left( \frac{A_t}{A_{t-1}} \right)^{1-\Psi} E_t \left[ \hat{V}_{t+1}^{1-\gamma} \right]^{1-\Psi}, \]

and it can be defined as an additional endogenous variable when solving the model.

Note that \( \hat{V}_t \) is homogeneous of degree \( \phi \) on consumption. Then the constant lifetime subsidy gives

\[ \hat{V}_{full} = (1 + \tau)^\phi \hat{V}_{partial} \]

or

\[ \left( \frac{\hat{V}_{full}}{\hat{V}_{partial}} \right)^{\frac{1}{\phi}} = 1 + \tau. \]

A.3 Solution

The stationary version of the model is solved by high-order perturbation method (order 3, implemented by Dynare v.4.4.2). The normalized value function \( \hat{V}_t \) is defined as an endogenous variable and its dynamic behavior is found as part of the solution of the model.

I simulate the model 3000 times, beginning from the deterministic steady state, and take the sample average of \( \hat{V}_1 \). This procedure delivers an approximation to the time-zero expected value \( E_0 \left[ \hat{V}_1 \right] \).
B Appendix: The decentralized economy

B.1 First-order conditions and log-linearized model

B.1.1 Household

The first order conditions associated with the problem of the household are:

$$C_t \cdot \frac{A_t}{a_{t-1}} = \phi_t \cdot \left( \frac{C_t}{A_t} - h \cdot \frac{C_{t-1}}{A_{t-2}} \right) - h \beta E_t \phi_{t+1} \cdot \left( \frac{C_{t+1}}{A_t} \cdot \frac{A_{t-1}}{A_{t-2}} - h \cdot \frac{C_t}{A_{t-1}} \right),$$

$$N_{i,t} : \quad E_{i,t} \left[ \frac{A_t}{A_{t-1}} \right] W_{i,t} = \psi N_{i,t},$$

$$\Lambda_t : \quad \frac{C_t}{A_{t-1}} - \frac{X_t}{A_{t-1}} - \frac{C}{1 + \sigma} \int_0^1 \left( U_{i,t+1} \cdot K_{i,t-1} \right) \cdot di - \tau \int_0^1 \left( \int_0^1 \left( B_{i,t-1} - 1 \right)^2 \cdot dj \div (\alpha_{i,t}^{\text{noise}}) \right) \cdot K_{i,t-1} \cdot di.$$ 

B.1.2 Firms

Let \( S(x) = \frac{\phi}{2} (x - 1)^2 \cdot x^{-1} \). The first order conditions associated with the problem of the firm can be expressed as

$$N_{i,t} \cdot \frac{W_{i,t}}{A_{t-1}} = \left( 1 - \alpha \right) \cdot \frac{X_{i,t}}{A_{t-1}},$$

$$U_{i,t} \cdot c U_{i,t} \cdot K_{i,t-1} \cdot \frac{A_{t-1}}{A_{t-1}} = \left( 1 - \delta \right) \cdot \frac{K_{i,t-1}}{A_{t-1}} + \chi_t \cdot \frac{X_{i,t}}{A_{t-1}} \left( \frac{X_{i,t}}{A_{t-1}} \cdot \frac{A_{t-2}}{A_{t-2}} \right),$$

$$\phi_{i,t} \cdot \left( \frac{A_t}{A_{t-1}} \right) \cdot \left( \frac{X_{i,t}}{A_{t-1}} \cdot \frac{A_{t-1}}{A_{t-1}} \right) = \left( \frac{X_{i,t}}{A_{t-1}} \cdot \frac{A_{t-1}}{A_{t-1}} \right) \cdot \left( \frac{X_{i,t}}{A_{t-1}} \cdot \frac{A_{t-1}}{A_{t-1}} \right) \div \left( \frac{X_{i,t}}{A_{t-1}} \cdot \frac{A_{t-1}}{A_{t-1}} \right).$$

$$X_{i,t} : \quad E_{i,t} \left[ \frac{A_t}{A_{t-1}} \right] \cdot \left[ \frac{A_t}{A_{t-1}} \right] = E_{i,t} \left[ \frac{A_t}{A_{t-1}} \right] \cdot \chi_t \left( 1 - S \left( \frac{X_{i,t}}{A_{t-1}} \cdot \frac{A_{t-2}}{A_{t-2}} \right) - S' \left( \frac{X_{i,t}}{A_{t-1}} \cdot \frac{A_{t-2}}{A_{t-2}} \right) \div \left( \frac{X_{i,t}}{A_{t-1}} \cdot \frac{A_{t-1}}{A_{t-1}} \right) \right],$$

$$K_{i,t} : \quad E_{i,t} \left[ \frac{A_t}{A_{t-1}} \right] = E_{i,t} \left[ \frac{A_t}{A_{t-1}} \right] \cdot \phi_{i,t+1} \cdot \left( \frac{A_{t+1}}{A_{t-1}} \right) \cdot \left( \frac{X_{i,t+1}}{A_{t-1}} \cdot \frac{A_{t-1}}{A_{t-1}} \right) \div \left( \frac{X_{i,t+1}}{A_{t-1}} \cdot \frac{A_{t-1}}{A_{t-1}} \right) \div \left( \frac{X_{i,t+1}}{A_{t-1}} \cdot \frac{A_{t-1}}{A_{t-1}} \right) \div \left( \frac{X_{i,t+1}}{A_{t-1}} \cdot \frac{A_{t-1}}{A_{t-1}} \right),$$

where \( \phi_{i,t} \) is the Lagrange multiplier with respect to the capital accumulation constraint.
B.1.3 Stock market

The demand for the stock satisfies the Euler equation:

\[ B_{j,t} : E_{j,t} \left[ \frac{A_{t}}{A_{t-1}^{\sigma}} \left( \frac{D_{i,t}}{A_{t-1}} - \frac{Q_{i,t}}{A_{t-1}} - \tau (B_{j,t} - 1) \frac{K_{i,t-1}}{A_{t-1}} \right) \right] = E_{j,t} \left[ \beta \frac{A_{t+1}^{\sigma} Q_{i,t+1}}{A_{t}} \left( \frac{A_{t}}{A_{t-1}} \right)^{1-\sigma} \right], \]

\[ \frac{D_{i,t}}{A_{t-1}} = \alpha \frac{Y_{i,t}}{A_{t-1}} - \frac{X_{i,t}}{A_{t-1}} - \frac{c}{1+\varsigma} U_{i,t}^{1+\varsigma} \frac{K_{i,t-1}}{A_{t-1}}. \]

B.1.4 Exogenous processes

The exogenous shocks satisfy

\[ \vartheta_{t} = \rho \vartheta_{t-1} + \varepsilon_{\vartheta}^{\vartheta}, \]
\[ \chi_{t} = \rho \chi_{t-1} + \varepsilon_{\chi}^{\chi}. \]

B.1.5 Log-linearized model

I log-linearize the first-order conditions around the deterministic steady state. Lowercase variables denote log-deviations from the steady state of the stationary uppercase variables.

After some algebra, the log-linearized model can be described as follows:

**Household:**

\[ (1 - h\beta) (1 - h) \lambda_{t} = (\vartheta_{t} - h\beta E_{t} \vartheta_{t+1}) (1 - h)^{-\sigma} \]
\[ -\sigma (c_{t} - h (c_{t-1} - \Delta a_{t-1})) + \sigma h\beta E_{t} (c_{t+1} - h (c_{t} - \Delta a_{t})), \]
\[ E_{i,t} \lambda_{t} + w_{i,t} = \eta n_{i,t}, \]
\[ y_{t} = \left( \frac{1-\beta(1-\delta)}{\alpha \beta} \frac{1-\alpha+\varsigma}{\varsigma} - \delta \right) c_{t} + \delta x_{t}. \]

**Firms:**

\[ w_{i,t} = y_{i,t} - n_{i,t}, \]
\[ u_{i,t} = \frac{y_{i,t} - k_{i,t-1}}{1+\varsigma}, \]
\[ y_{i,t} = \alpha (u_{i,t} + k_{i,t-1}) + (1 - \alpha) (\Delta a_{t} + \xi_{i,t} + n_{i,t}), \]
\[ k_{i,t} + \Delta a_{t} = (1 - \delta) k_{i,t-1} + \delta x_{i,t}, \]
\[ \phi_{i,t} = E_{i,t} [ -\sigma \Delta a_{t} + (1 - \beta (1 - \delta)) (\lambda_{t+1} + y_{i,t+1} - k_{i,t}) + \beta (1 - \delta) \phi_{i,t+1} ]; \]
\[ E_{i,t} \lambda_{t} = \phi_{i,t} + \chi_{t} - \phi (x_{i,t} - x_{i,t-1} + \Delta a_{t-1}) + \beta \phi E_{i,t} (x_{i,t+1} - x_{i,t} + \Delta a_{t}). \]
Stock market:

\[ d_{i,t} = \frac{((1 - \beta (1 - \delta))) y_{i,t} - \beta \delta x_{i,t}}{1 - \beta}, \]
\[ b_{j,t} = \frac{1}{\tau \beta} ((1 - \beta) d_{i,t} - q_{i,t} + \beta E_{j,t} [(\lambda_{t+1} - \lambda t + q_{i,t+1} + (1 - \sigma) \Delta a_t)]), \]
\[ q_{i,t} = (1 - \beta) d_{i,t} + \beta \int_0^1 E_{j,t} [(\lambda_{t+1} - \lambda t + q_{i,t+1} + (1 - \sigma) \Delta a_t)] dj + \beta \tau e_{i,t}^{\text{noise}}. \]

Market clearing:

\[ x_t = \int_0^1 x_{i,t} di, \]
\[ y_t = \int_0^1 y_{i,t} di. \]

B.2 Evolution of beliefs

B.2.1 Updating the common beliefs

The productivity process has the VAR representation

\[ X_t = F X_{t-1} + e_t, \]

where

\[ X_t = \begin{bmatrix} z_t \\ v_t \\ z_{t-1} \end{bmatrix}, \]
\[ e_t = \begin{bmatrix} \varepsilon_t \\ \mu_t \\ 0 \end{bmatrix}, \]
\[ F = \begin{bmatrix} 1 + \rho_z & 0 & -\rho_z \\ 0 & \rho_v & 0 \\ 1 & 0 & 0 \end{bmatrix}. \]

Let

\[ Y_t = \begin{bmatrix} a_t \\ s_t^f \end{bmatrix}, \]

denote the vector of aggregate signals. It satisfies the observation equation

\[ Y_t = H' X_t + u_t, \]
where
\[
H' = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix},
\]
\[
u_t = \begin{bmatrix} 0 \\ e'_{t} \end{bmatrix}.
\]

Let \(Y_t = \{Y_t, Y_{t-1}, Y_{t-2}, \ldots\}\) denote the history of aggregate signals up to period \(t\). Define
\[
X_{t|t} = E[X_t|Y_{t-1}].
\]

The algebra of the stationary Kalman filter (e.g., equations 13.2.15 and 13.5.3 from Hamilton) delivers
\[
X_{t|t} = WX_{t-1|t-1} + KY_t,
\]
where \(W = (I - KH')F\), \(K = \Sigma H (H'\Sigma H + R)^{-1}\), \(R = Var[u_t]\), and \(\Sigma\) is the stationary variance associated with the filter. The matrix \(\Sigma\) satisfies
\[
\Sigma = F \left[ \Sigma - \Sigma H (H'\Sigma H + R)^{-1} H'\Sigma \right] F' + Q,
\]
where \(Q = Var[e_t]\). These equations characterize the evolution of the beliefs of the consumer.

**B.2.2 Evolution of agent-specific beliefs**

Assume that an economic agent holds the prior belief \(X_{t-1|t-1}\) and observes the signal \(\tilde{Y}_t = \tilde{H}'X_t + w_t\) (not necessarily equal to the aggregate signal \(Y_t\), the signals used up to \(t - 1\)). Note that
\[
E\left[\tilde{Y}_t|Y_{t-1}\right] = \tilde{H}'X_{t|t-1}
\]
with error
\[
\tilde{Y}_t - E\left[\tilde{Y}_t|Y_{t-1}\right] = \tilde{H}' \left(X_t - X_{t|t-1}\right) + w_t
\]
and MSE
\[
\tilde{H}'\Sigma\tilde{H} + S
\]
where \(S = Var[w_t]\).

The inference about \(X_t\) is updated on the basis of the observation \(\tilde{Y}_t\) to produce \(\tilde{X}_{t|t} = E\left[X_t|\tilde{Y}_t, Y_{t-1}\right]\). This can be evaluated using the formula for updating a linear projection,
\[
\tilde{X}_{t|t} = X_{t|t-1} + \left\{ E\left[\left(X_t - X_{t|t-1}\right) \left(\tilde{Y}_t - E\left[\tilde{Y}_t|Y_{t-1}\right]\right)\right]\right\} \\
\times \left\{ E\left[\left(\tilde{Y}_t - E\left[\tilde{Y}_t|Y_{t-1}\right]\right) \left(\tilde{Y}_t - E\left[\tilde{Y}_t|Y_{t-1}\right]\right)\right]\right\}^{-1} \times \left(\tilde{Y}_t - E\left[\tilde{Y}_t|Y_{t-1}\right]\right).
\]
But we know
\[
E \left[ \left( X_t - X_{t|t-1} \right) \left( \bar{Y}_t - E \left[ \bar{Y}_{t|Y_{t-1}} \right] \right) \right] = E \left[ \left( X_t - X_{t|t-1} \right) \left( \bar{H}' \left( X_t - X_{t|t-1} \right) + w_t \right) \right] = \Sigma \bar{H}.
\]

Substituting,
\[
\bar{X}_{t|t} = X_{t|t-1} + \Sigma \bar{H} \left( \bar{H}' \Sigma \bar{H} + S \right)^{-1} \left( \bar{Y}_t - \bar{H}' X_{t|t-1} \right).
\]

Finally, since \( X_{t|t-1} = FX_{t-1|t-1} \),
\[
\bar{X}_{t|t} = FX_{t-1|t-1} + \Sigma \bar{H} \left( \bar{H}' \Sigma \bar{H} + S \right)^{-1} \left( \bar{Y}_t - \bar{H}' FX_{t-1|t-1} \right)
= \bar{W} X_{t-1|t-1} + \bar{K} \bar{Y}_t,
\]
where \( \bar{K} = \Sigma \bar{H} \left( \bar{H}' \Sigma \bar{H} + S \right)^{-1} \) and \( \bar{W} = \left( I - \bar{K} \bar{H}' \right) F \).

This formula allows to update for the (interim) expectations of firms and traders, by defining the appropriate matrices \( \bar{H} \), \( S \) and orthogonal shocks \( w_t \).

### B.3 Data appendix

All data are quarterly.

**Consumption** (\( C_t \)) Real consumption per capita, quarterly, chained dollars 2009, from National Income and Product Accounts (NIPA), as reported by the Bureau of Economic Analysis (BEA), downloaded from the Federal Reserve Economic Data (FRED). Defined as Real personal consumption expenditures per capita (A794RX0Q048SBEA) minus Real personal consumption expenditures per capita in Durable goods (A795RX0Q048SBEA).

**Investment** (\( X_t \)) Real investment per capita. Recovered from FRED, as reported by BEA. Defined as Real Gross Private Domestic Investment (GPDIC96) divided by Population (midperiod) plus Real personal consumption expenditures per capita in Durable goods.

**Output** (\( Y_t \)) Real GDP per capita. Defined as Real Gross Domestic Product (GDPC96) divided by Population (midperiod).

**Labor** (\( N_t \)) Hours worked. Defined as the Average Weekly Hours in the Nonfarm Business Sector (PRS85006023) minus a linear trend in time. Average hours are detrended because in the model this variable is stationary but it presents a secular trend in the data.

**Asset price index** (\( Q_t \)) Real S&P500 index. Quarterly, end of period, recovered from The Center for Research in Security Prices (CRSP). Defined as S&P500 divided by the GDP price deflator from NIPA (GDPDEF).
TFP ($A_t$) Quarterly, utilization-adjusted series on TFP, constructed by Fernald [2012]. Series dtfp_util.

Firm-level productivity Persistence and variance of the firm-specific component of productivity, estimated in Imrohoroglu and Tuzel [2014].


Profits growth forecasts ($E_{j,t} [\Delta d_{t+1}]$) Forecast on corporate profits after taxes, from the Survey of Professional forecasters.

Cross-sectional volatility ($\sigma(\cdot)$) Dispersion Measure D2 = 75th Percentile Minus 25th Percentile of the Forecasts for Q/Q Growth. In the model, the cross-sectional distribution of forecasts is Normal. The Measure D2 corresponds to $1.3490 \times \sigma$, where $\sigma$ is the standard deviation of the corresponding normal distribution.

B.4 Calibration

Aggregate productivity First, note that the theoretical moments of the log-growth rate of productivity are:

$$Var[\Delta a_t] = \frac{\sigma_\varepsilon^2}{1 - \rho_z^2} + \frac{2}{1 + \rho_v} \frac{\sigma_\mu^2}{1 - \rho_v^2}$$

$$Cov[\Delta a_t, \Delta a_{t-j}] = \frac{\rho_z^j}{1 - \rho_z^2} \sigma_\varepsilon^2 - \rho_v^{j-1} \frac{1 - \rho_v}{1 + \rho_v} \sigma_\mu^2.$$ 

A proof for this fact can be found in Blanchard et al. [2013].

I use the quarterly, utilization-adjusted series on TFP (Fernald [2012], 1947:Q2-2014:Q1) and match the sample variance and autocovariances of the log-growth TFP with the theoretical moments. In particular, from the data,

$$\sigma(\Delta a_t) = 0.0084, \quad \rho(\Delta a_t, \Delta a_{t-1}) = -0.0952,$$

$$\rho(\Delta a_t, \Delta a_{t-2}) = -0.0055, \quad \rho(\Delta a_t, \Delta a_{t-3}) = 0.0254.$$ 

From this procedure, I obtain

$$\rho_v = 0.4526, \quad \rho_z = 0.7414,$$

$$\sigma_\varepsilon = 0.0024, \quad \sigma_\mu = 0.0065.$$ 

Noisy trade and portfolio adjustment cost The scale parameter on the portfolio adjustment cost, $\tau$, is not separately identified in the linear solution of the log-linearized model.
The Simulated Method of Moments allows to identify the product $\tau \times \sigma_{\text{noise}}$, and in the baseline calibration, this product takes the value $4.3938 \times 10^{-4}$.

Following Hassan and Mertens [2014], I calibrate $\tau$ to satisfy

$$
\sigma(B_{i,\text{noise}}) = 0.0043 \times Y,
$$

where $Y$ is the deterministic steady-state value of output, and $B_{i,\text{noise}} = \exp(e_{i,t}^{\text{noise}}) - 1$ is the noisy trade in sector $i$. The formula for the variance of a log-normal random variable gives us

$$
\sigma(B_{i,\text{noise}}) = \exp(\sigma_{\text{noise}}^2) \left( \exp(\sigma_{\text{noise}}^2) - 1 \right).
$$

In steady state, $Y = 1.0332$ and solving for $\sigma_{\text{noise}}^2$, I get

$$
\sigma_{\text{noise}} = 0.0044 \quad \text{and} \quad \tau = 0.0996.
$$

### B.5 Alternative information structure

Under the alternative information structure, the current level of aggregate productivity, $a_t$, is commonly observed. The stock price is a signal only about the permanent component, $z_t$. The model is recalibrated for the alternative information structure.

#### Table 5: Annualized consumption compensation (%)

<table>
<thead>
<tr>
<th>No dispersed</th>
<th>No noise</th>
<th>No fin. shock</th>
<th>Dispersed</th>
<th>No signal</th>
<th>Common</th>
<th>Full info</th>
<th>No info</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0001</td>
<td>-</td>
<td>0.0009</td>
<td>0.0002</td>
<td>-0.0106</td>
<td>0.0002</td>
<td>0.0926</td>
<td>-0.0106</td>
</tr>
</tbody>
</table>

No dispersed information: $\sigma_{\text{disp}} = 0$; No noise traders: $\sigma_{\text{noise}} = 0$; No financial shock: $\sigma_f = 0$. Dispersed: $\sigma_{\text{disp}} \to \infty$; No signal: $\sigma_f \to \infty$.

Common information: the aggregate signals $a_t$ and $s^f_t$ are observed by all agents.

Full information: common information, and $s^f_t = z_t$, the permanent component productivity is commonly observed.

No information: common information, no signals about $z_t$.

### B.6 Welfare criterion

Let

$$
W_t = \left( C_t - hC_{t-1} \right)^{1-\sigma} - \frac{\psi}{1+\eta} A_{t-1}^{1-\sigma} \int_0^1 N^{1+\eta}_{i,t} di + \beta E_{t} W_{t+1}
$$

In the linear equilibrium, $n_{i,t} \approx \ln N_{i,t} - \ln N$, follows a linear function that depends on past aggregate variables and current Gaussian innovations. In a stationary equilibrium, the cross-sectional distribution of labor effort, $n_{i,t} \sim N(n_t, \sigma_n^2)$, where $\sigma_n^2$ denotes the cross-sectional
variance. Then,
\[
\int_0^1 N_{i,t}^{1+\eta} di = N^{1+\eta} \int_0^1 e^{(1+\eta) n_{i,t}} di = N^{1+\eta} e^{(1+\eta) \int_0^1 n_{i,t} di + \frac{1}{2} (1+\eta)^2 \sigma_n^2} = N^{1+\eta} e^{(1+\eta) n_t + \frac{1}{2} (1+\eta)^2 \sigma_n^2}.
\]

Also, from the linearized solution,
\[
\left( \frac{C_t}{A_{t-1}} - h \frac{C_{t-1}}{A_{t-2}} \frac{A_{t-2}}{A_{t-1}} \right)^{1-\sigma} = C^{1-\sigma} \left( e^{ct} - h e^{c_{t-1} - \Delta a_{t-1}} \right)^{1-\sigma},
\]
and then the exact instantaneous utility, evaluated under the linear approximation decision rules, can be found. Finally, the normalized (stationary) welfare is
\[
\hat{W}_t = \frac{1}{1-\sigma} C^{1-\sigma} \left( e^{ct} - h e^{c_{t-1} - \Delta a_{t-1}} \right)^{1-\sigma} - \frac{\psi}{1+\eta} N^{1+\eta} e^{(1+\eta) n_t + \frac{1}{2} (1+\eta)^2 \sigma_n^2} + \beta e^{(1-\sigma) \Delta a_t} E_t \hat{W}_{t+1},
\]
where \( \hat{W}_t = \frac{W_t}{A_{t-1}^{\sigma}} \).

I find an approximation for this object using a high-order perturbation method (3rd order in Dynare v.4.4.2).