1 Triangles on Earth

We all know what happens if I draw a triangle on a flat piece of paper. But what happens if I draw a triangle on a curved surface, like the Earth?

I’ve cheated, most of you say, because that’s not a triangle! The lines are curved, and a triangle should have straight sides. Well, really I haven’t Take a look at the same Earth, but at points just above each of the corners:

![Figure 1: Earth, seen from space, with a big triangle over North America.](image)

![Figure 2: The same triangle, viewed from the three vertices](image)
1. Looking at Figure 2, what are the three angles of the triangle?
   - \( \angle A = \)
   - \( \angle B = \)
   - \( \angle C = \)

2. What is the sum of the three angles?
   \[ \angle A + \angle B + \angle C = \]

What’s going on? On a sphere, the angles of a triangle can add up to **more than 180°**. The sum of angles of a triangle depends on the area of the triangle, and the radius of the sphere:

\[ \angle A + \angle B + \angle C = 180^\circ \cdot \left(1 + \frac{\text{Area}}{\pi \cdot \text{Radius}^2}\right) \]  

(1)

4. Can you guess, by looking at the maps, what **Fraction** of the Earth the triangle covers? Circle your answer.

   Fraction = \( \frac{1}{16} \), \( \frac{1}{8} \), \( \frac{1}{4} \), \( \frac{1}{2} \)

5. If we rewrite the sum of angles equation as

\[ \angle A + \angle B + \angle C = 180^\circ \cdot (1 + 4 \cdot \text{Fraction}) \]  

(2)

what does the right hand side equal?

6. Is this the same answer you got for question 2?
2 Triangles on Maps

Figure 3: The Mercator Projection of Earth, where the North and South Poles get split apart. Lines and angles drawn on this map are the same as they are on the sphere, so the lines look straight, but the angle at the North Pole gets ripped apart.

Figure 4: The Mollweide Projection of Earth, which you get by pretending the Earth is a cylinder. The areas of countries look better, but the angles are all wrong.
2.1 Questions

1. Imagine I have a huge sphere with a tiny triangle. In this case, the area of the triangle is $A = 100m^2$, about the size of a map, and it’s on a sphere with a radius of 6,000,000m (about the size of Earth).

   • What equation should I use for this, equation (1) or equation (2)?

   • What will the sum of the three angles of the triangle be?

   • Is that very different from what the triangle would look like if the Earth were flat?

3 Triangles in Space

Figure 5: The Mollweide Projection of the Cosmic Microwave Background, the light left over from the Big Bang. By looking for triangles on the sphere, astronomers can try to figure out if the Big Bang created a universe curved like a sphere, or flat like a piece of paper.
3.1 Questions

1. Now imagine I have a ridiculously large sphere, with a radius of \(4.6 \times 10^{10}\) lightyears. This is roughly how far away the picture of the universe in Figure 5 was taken from. Say I build a giant equilateral triangle in space, to measure the angles and see if the universe is curved.

![Figure 6: My imaginary triangle in space.](image)

- If the universe is flat, and the triangle is equilateral, what should the three angles equal?
  \[\angle A = \angle B = \angle C = ?\]

- If the universe were really curved, would you expect the angles to be bigger or smaller than your answer?

- **Bonus Question** - In order to be sure, I want to build a triangle big enough that the angles would be 61° (if the universe looked like a sphere). How big would the area need to be for me to have \(\angle A = \angle B = \angle C = 61°\)? Use the equation

  \[3 \cdot \angle A = 180° \left(1 + \frac{\text{Area}}{\pi \cdot \text{Radius}^2}\right)\]